

# **Modelling the Spread of HIV/AIDS: An application to Bauchi and Gombe State Nigeria**

Oyelami<sup>1</sup>, B.O.; Ogidi<sup>2</sup>, J. A.; Onumanyi<sup>3</sup>, P.; Yacouba<sup>1</sup>, M.

1. *Mathematical Sciences Programme, ATBU, Bauchi, Nigeria*
2. *Biological Sciences Programme, ATBU, Bauchi, Nigeria.*
3. *Mathematics Department, University of Jos, Nigeria.*

## **Abstract**

HIV-1 has been found to be responsible for the spread of HIV/AIDS in West Africa. The Mathematical model for the transmission dynamics of the HIV-1 in a heterosexual population in BAUCHI and GOMBE States stratified by age, sex and sexual activity was simulated on the computer and results obtained.

The research emphasis is on the demographic and epidemiological impacts of aids in the States.

The results obtained for Males when compared with females especially at the age of 15 years and above, show a dramatic increase as a result of the demand for sex at this age group. The results are exceptionally higher for males.

The overall cost that is needed to put the aids patients on life supporting drugs (e.g. Zidovudine) when computed on monthly and annual basis and the overall cost from the stage acquisition of the disease to the terminal stage for the patients were computed. The computed result indicated that in the year 2003 about 1266 Aids patients would be recorded in the States and ₦1.00267 billion would be needed to support the patients on drugs; such colossal sum will be needed to support less than 0.1% of the state's population (1266 patients). The sum is so huge to be borne by any state in Nigeria.

It is thus concluded that if the current trend is unchecked, a catastrophic AIDS epidemic will occur in the near future in the states.

AMS 1998: 65C, 92C

## ***Keywords or phrases***

HIV, AIDS, deterministic model, spectrum and drug administration.

## 1. Introduction

Acquired Immune Deficiency Syndrome (AIDS) is a disease caused by the Human Immunodeficiency Virus (HIV) and two of its types have been identified; these are: HIV-1 and HIV 2; a recent study has revealed that HIV-1 is prevalent and HIV-2 is commonly found in the West Africa.

AIDS was first reported in USA in 1981 and two years later HIV was found to be responsible for the disease (Anthony, 1989). It is found to be predominantly transmitted through sexual intercourse and blood transfusion.

Factors that enhance the spread of the disease are known to be increasing urbanization, migrant labour, expanding tourist travel and changing attitude towards sexual behaviour in the modern society and lack of proper health management policy.

AIDS has become one of the most dreaded diseases of mankind (Geoffrey et al., 1994; Anderson et al., 1991, 1992). The World Health Organization (WHO) has predicted that 27.8 million people will have HIV infection globally in the mid 1996, of which 63% will be Africans. Even though AIDS is known to be prevalent in Nigeria, there is no adequate statistics on it and no attempt to model the spread.

This paper seeks to: develop a model for the spread of HIV and forecast the extent of spread in the near future as well as compute the cost of maintaining the victims on a life supporting drug.

## 2. Methodology

Data of the spread of AIDS is obtained from the old BAUCHI State Health Management Board on the occurrence of the disease from 1984 to 1997. The information on reported cases of the disease is contained in table (1). From the information in the table; it appears the number of cases was under-reported. We are then left to decide how to take care of the shortfall or to simply use the statistical information available. The latter option was chosen in the absence of any other reliable method of estimating the shortfall. The progressive occurrence of the disease was then generated on the basis of the available record using the deterministic mathematical models. Numericalisation of model was done to extrapolate the number of occurrence cases from 1995 - 2003. The cost of life supporting drugs for the projected patients from the year 1994 to 2003 was also computed.

## 3. The Mathematical Model of the HIV/AIDS

The following deterministic model can represent the transmission of HIV/AIDS:

$$\frac{\partial X_{KI}(a,t)}{\partial a} + \frac{\partial X_{KI}(a,t)}{\partial t} = - [\lambda_{KI}(t) + \mu_K(a)] X_{KI}(a,t) \quad (1)$$

$$\frac{\partial Y_{IKl}(a,t)}{\partial a} + \frac{\partial Y_{IKl}(a,t)}{\partial t} = \lambda_{Kli}(t) X_{Kl}(a,t) \quad (2)$$

$$= -[\gamma_i(a) + \mu_k(a)] Y_{SkI}(a,t)$$

$$\frac{\partial A_{Kl}(a,t)}{\partial a} + \frac{\partial A_{Kl}(a,t)}{\partial t} = \gamma_s(a) Y_{SkI}(a,t) \quad (3)$$

$$= -[\mu_K(a) + \alpha_K(a)] A_k(a,t)$$

$$\frac{\partial Y_{SkI}(a,t)}{\partial a} + \frac{\partial Y_{SkI}(a,t)}{\partial t} = \gamma_{s-1}(a) Y_{(s-1)kI}(a,t) \quad (4)$$

$$= -[\gamma_s(a) + \mu_K(a)] Y_{SkI}(a,t)$$

$$C(t) = \sum_{k=1}^n A_{Kl}(a,t) d(t) \quad (5)$$

where

$X_{Kl}(a,t)$  Is the number of susceptible people to the HIV disease of sex k and age a in the activity group of l at time t.

$Y_{SkI}(a,t)$  Is the number of people infected and infectious but without disease symptoms and s is the stage (s = 1, 2, 3).

$$A_{Kl}(a,t)$$

Number of people affected by AIDS at k age group in the l activity group  $\mu_k(a)$  is the age and sex specific death rate.

$\gamma_s(a)$  is the stage specific progressing from s infectious stage to full-blown AIDS.

$\lambda_{Kli}(t)$  is the rate per annum at which susceptible acquire the HIV disease.

$C(t)$  is the cost evaluation of the drugs needed to put the AIDS patient on life supporting drugs.

$d(t)$  is the cost of procurement of the drug.

#### 4. Numericalization of the Model

Let the  $(x,t)$  plane be divided into a network of rectangles of sides  $\Delta x = h$  and  $\Delta t = k$  by drawing the set of lines  $a_i = a_0 + ih, i = 0, 1, 2, \dots, N - 1$  and  $t_j = t_0 + jk, j = 0, 1, 2, \dots, N - 1$ .

Define

$$X_{Kl}(a,t) = X_{i,j}^{Kl} = X_{Kl}(a_0 + ih, t_0 + jk)$$

$$Y_{ikl}(a, t) = Y_{i,j}^{ikl} = Y_{ikl}(a_o + ih, t_o + jk)$$

$$Y_{skl}(a, t) + Y_{i,j}^{skl} = Y_{ikl}(a_o + ih, t_o + jk)$$

$$A_{kl}(a, t) = Y_{i,j}^{skl} = A_{i,j}^{ki}(a_o + ih, t_o + jk)$$

Thus, if we replace the derivatives in eqs (1-4) in section 3 by backward difference formula for derivative of a, and let

$$X_{i,j}^{kl} \approx X_i, Y_{i,j}^{ikl} \approx Y_i, Y_{i,j}^{skl} \approx Y_i^s, A_{i,j}^{ks} \approx A_i.$$

Then equations (1-4) become

$$\frac{dX_i(t)}{dt} + \frac{1}{k}(X_i(t) - X_{i-1}(t)) = -[X_i^{kl} + \mu^k] X_i(t) \quad (5)$$

$$\frac{Y_i(t)}{dt} + \frac{1}{k}(Y_i(t) - Y_{i-1}(t)) = \lambda^{kli} X_i(t) - [\gamma_i + \mu^k] Y_i(t) \quad (6)$$

$$\frac{dY_i^s(t)}{dt} + \frac{1}{k}(Y_i^s(t) - Y_{i-1}^s(t)) = \gamma_{(s-1)}(a) Y_i^{s-1} - [\gamma_s - \mu^k] Y_i^{skl} \quad (7)$$

$$\frac{dA_i(t)}{dt} + \frac{1}{k}(A_i(t) - A_{i-1}(t)) = \gamma_s Y_i^{ske} - [\mu^k + \alpha^k] A_i(t) \quad (8)$$

$$I = 1, 2, \dots, N-1.$$

These equations (5-8) are equivalent to the following eqs(9-12), respectively

$$\frac{dX(t)}{dt} = -C_1(t)X(t) + b_1(t) \quad (9)$$

$$\frac{dY(t)}{dt} = -C_2(t)Y(t) + \lambda(t)X(t) + b_2(t) \quad (10)$$

$$\frac{dY^s(t)}{dt} = -C_3(t)Y^s(t) - \phi(t)A(t) + b_3(t) \quad (11)$$

$$\frac{dA(t)}{dt} = C_4(t)A(t) + \gamma_s(t)Y^s(t) - b_4(t) \quad (12)$$

$$C_i(t) = \frac{1}{k} \begin{bmatrix} 1-k\phi_i(t) & 0 & 0 & \dots & 0 \\ 0 & 1-k\phi_i(t) & 0 & \dots & 0 \\ 0 & 0 & 1-k\phi_i(t) & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 1-k\phi_i(t) \end{bmatrix}$$

... -1 1 1-kφ<sub>i</sub>(t) , i = 1, 2,...4.

$$\phi_1(t) = 1 + X_l^{ki} + \mu_k \quad (13)$$

$$\phi_2(t) = I + \gamma_i + \mu_k$$

$$\phi_3(t) = I + \gamma_s + \mu_k$$

$$\phi_4(t) = I + \mu^k + \alpha_k$$

$$X(t)=(X_i(t)), Y(t)=(Y_i(t)), A(t)=(A_i(t))^T,$$

$$Y^s(t)= (Y_i^s(t))^T, = 1,2,...N-1$$

b<sub>i</sub>(t) = [b<sub>i</sub>(t)/h, 0, 0, ... 0] are known from the boundary condition. Whenever b<sub>i</sub>(t) are constant vectors, the solution of equation (9) is given by

$$X(t) = b(t) \exp\left(\int_0^t -C_1(s) ds\right) + \int_0^t b(\tau) \exp\left(\int_0^{k+s} b-C(s) ds\right) d\tau \quad (14)$$

and satisfies the recurrence relation

$$V(t+k) = \exp\left(-\int_0^k C(s) ds\right) V(t) + A(k) \quad (15)$$

Using the (s, t) Pade' s approximation to exp (θ)

That is,

$$\exp(\theta) = \frac{I + P_1\theta + P_2\theta^2 + \dots + P_t\theta^t}{I + q_1\theta + q_2\theta^2 + \dots + q_s\theta^s} + C_{s+t+1}(\theta) \quad (16)$$

$$+ O(\theta^{s+t+s})$$

C<sub>s+t+1</sub> = constant, P<sub>i</sub>, q<sub>i</sub>, we undetermined coefficients for I = 1, 2, ..., s, t.

Replacing the exponential expression in equation(14) by the (1,0) Pade's approximation for

$$\exp\left(-\int_0^k C(s) ds\right)$$

We get

$$\exp\left(-\int_0^k C(s) ds\right) \approx \left(I + \int_0^k C(s) ds\right)^{-1} \quad (17)$$

Where

I is n x n identity matrix

C(t) is n x n matrix defined in time t

The principal error term for the approximation in equ (28) is

$$\left( \int_0^k C^2(s) ds \right)^{-1}$$

That is the inverse of the matrix  $\int_0^k C^2(s) ds$ .

Therefore, the numerical solution to the HIV/Aids model can be generated from the following iterative equation:

$$X_{j+1} = b_k^1 (I - \int_0^k C_1(s) ds)^{-1} X_j + H_j \quad (18)$$

$$Y_{j+1} = b_k^2 (I - \int_0^k C_2(s) ds)^{-1} Y_j + N_j \quad (19)$$

$$Y_{j+1}^s = b_k^3 (I - \int_0^k C_3(s) ds)^{-1} Y_j^s + M_j \quad (20)$$

$$A_{j+1} = b_k^4 (I - \int_0^k C_4(s) ds)^{-1} A_j + Z_j, j = 0, 1, 2, \dots, N-1 \quad (21)$$

where

$$H_j = (I - \int_0^j C_1(s) ds)^{-1} \left[ \int_0^j b_1(s) (I - \int_0^k C_1(\tau) d\tau)^{-1} ds \right] \quad (22)$$

$$N_j = (I - \int_0^j C_2(s) ds)^{-1} + \int_0^j [(\lambda(s) H_j + b_2(s) ds (I - \int_0^r C_2(s) ds)] d\tau \quad (23)$$

$$M_j = (I - \int_0^j C_3(s) ds)^{-1} + \int_0^j [-\phi(s) N_j - b_3(s) ds (I - \int_0^r C_3(s) ds)] d\tau \quad (24)$$

$$Z_j = (I - \int_0^j C_4(s) ds)^{-1} + \int_0^j [(\gamma_s(s) M_j - b_4(s)) ds (I - \int_0^r C_4(s) ds)] d\tau \quad (25)$$

$$\frac{\text{Max}}{i} (I - \int_0^t \lambda(C_i(s)) ds)^{-1} < 1, i = 1, 2, \dots, N-1. \quad (26)$$

where  $\lambda_i(C_i(s))$  are the eigenvalues of the matrix  $C_i(t)$ .

N is suitably chosen to allow the convergence of the iterative scheme to the solution without giving rise to complexity that are often associated with the iterative scheme which occupies a large memory space. For definitiveness, we allow  $N \leq 10$ .

## 5. Result

The algorithm in equation (18-25) was simulated on the computer under the given stability criteria in equation (26), using the following data (see table 1) obtained on the spread of Aids in the given states:

Disease	Year	Under 1 year	5-15 years	15 years	under 1 year	5-15 years	15 & above	Total
AIDS	1994			11				15
	1995			18				26
	1996			21			10	31
	1997			28			15	43

Table 1: BAUCHI and GOMBE States AIDS recurrence cases data for 1994 - 1997

Source: The Old BAUCHI State's Health Management Board.

---

$\beta_{sk}$	Transmission probability (per partnership) from men to women		0.2
$\gamma_s$	Parental (pre-natal)	1	0.35
$\gamma_s(a)$	Rate of transition between infections states (per capital per year)		
	for age < 15 years (S = 1, 2, 3)		1.5
	for age > 15 years (S = 1)		1.0
	for age > 15 years (S = 2)		1666
	for age > 15 years (S = 3)	1.0	
$\alpha(a)$	AID - associated death rate (per capital per year)		1.0

---

Table 2: The parameter used for the computer simulation.

Source: Geoffrey and Roy, (1994).

The computerized result as obtained from the model is displayed in table 3 as follows:

Disease AIDS	Year	Male			Female			Total for AIDS
		$Y_{SkI}$	$A_{Kl}$	$S_{Kl}$	$Y_{SkI}$	$A_{Kl}$	$S_{Kl}$	
	1998	121.8	30	273	87.50	17.0	183.00	47
	1999	234.7	90	274.00	175.0	21.0	188.00	114
	2000	487.5	380	273.00	350	77	188.05	457
	2001	975	390	273.01	700	420	190.00	810
	2002	1950	409	273.07	1400	450	193.04	859
	2003	3900	656	273.02	2800	610	183.05	1266

Table 3: Computerized result as obtained from the model for 1998 - 2003.

The cost evaluation to put the affected AIDS patients on life supporting drugs namely Zidovudine and Lavirage are calculated. These drugs are known to extend the life survival of aids patients for approximately 12 - 18 months. It has been estimated that the cost of the drugs in a year is ₦66,000 or roughly ₦10 million for patient from period of diagnosis to death (Stopaid, 1998) base on this information, tables 3 and 4 compute the cost evaluation for 1994 - 1997 as obtained from table 1 and for 1998 - 2003 as obtained from table 3. The cost evaluation is made in three forms: monthly cost, annual cost and the overall cost.

Year	1994	1995	1996	1997
Monthly cost in million	8.33	22.50	17.22	23.889
Annual cost for the individual	99.96	270.00	206.64	286.67
Overall cost	150	270.00	310.00	430.00

Table 4: Cost evaluation for drug therapy as obtained from table 1.

--	--	--	--	--	--	--

Year	11998	11999	22000	22001	22002	22003
Monthly Cost	26.11	63.33	253.89	450.00	477.22	703.33
Annual Cost	313.32	759.96	3046.68	54.00	5726.64	8439.96
Overall Cost	470	1114	4570	8010	8590	12660

Table 5: Cost evaluation for drug therapy as obtained from Table 2.

Fig. (1 - 4) shows the graphical display of the computer output for the simulated model as obtained from data in tables 1, 3, 4, and 5. Three dimensional graphical display for the model along with the drugs cost distribution contour and 3D plot population distribution for male and female for YSA patients are also included.

## Discussion

The emphasis on this work is on demographical impact of Aids on the society, our discussion centers on computer output for Aids especially. The computer output for the model shows that the Aids disease progressively increases with years and very endemic for ages of 15 where it is observed that the demand for sex is higher at this age group, especially for males.

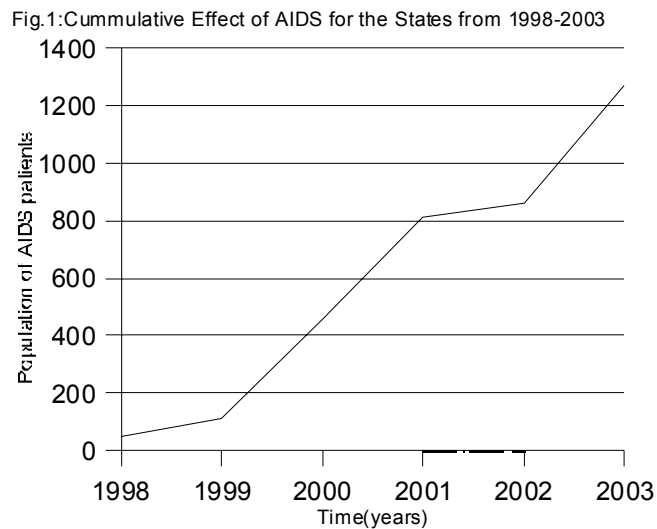


Fig.3:YSA patients'plot obtained from Table 3.

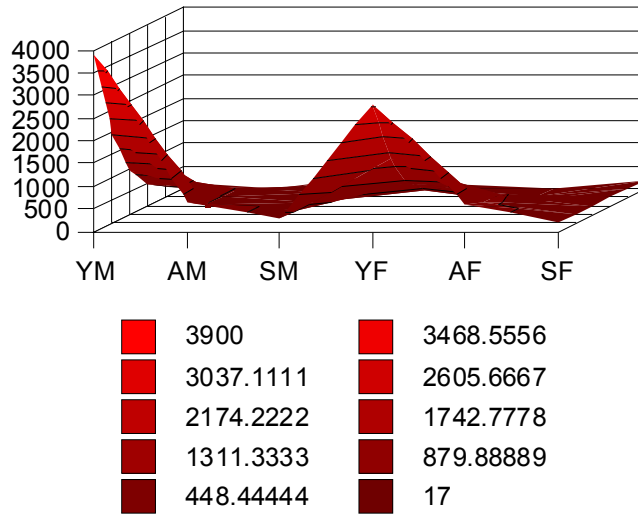


Fig.4:Drug Administration contour

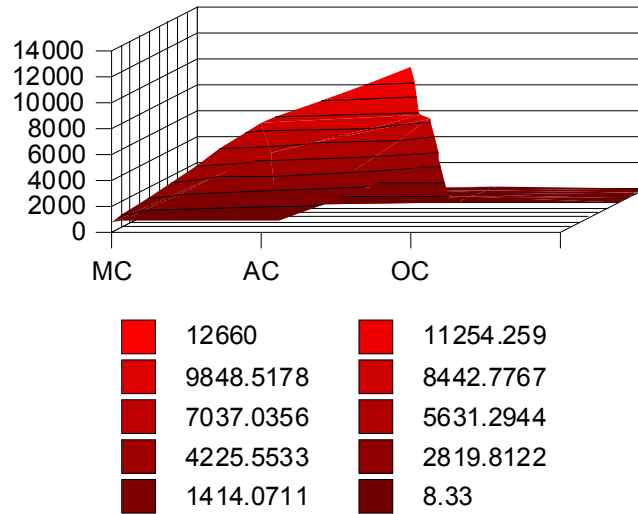


Fig 5:The recurent case of AIDS from 1994-2003

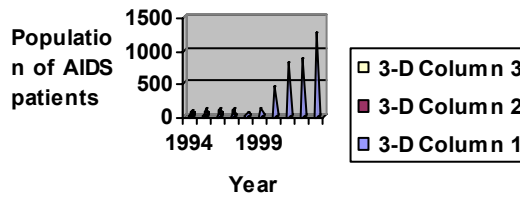


Fig.6: 3D plot for AIDS disease obtained from Table 3  
Population of Female AIDS Patients

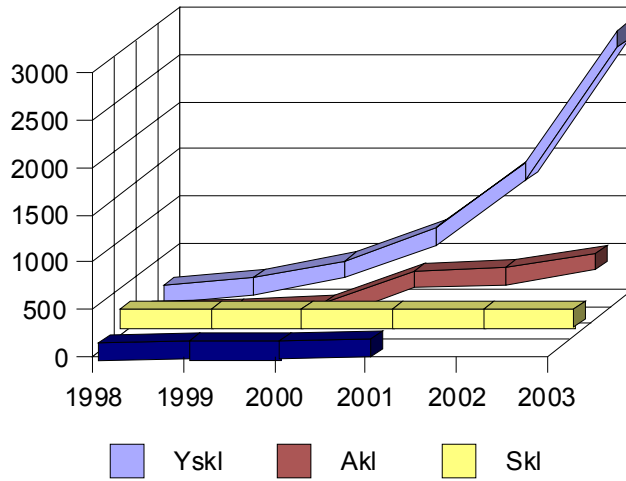
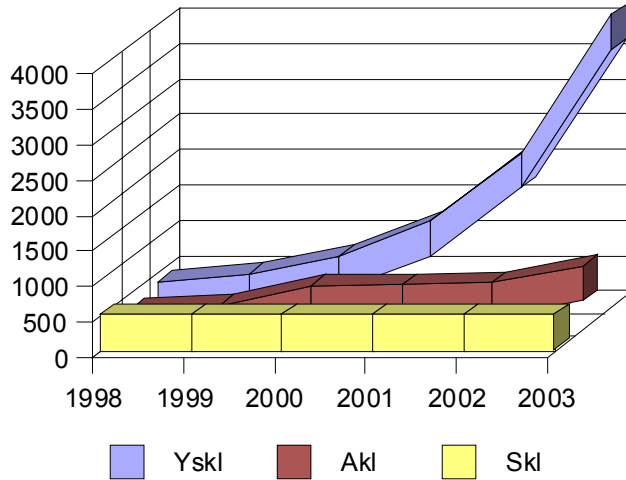


Fig. 7: 3D plot for the AIDS disease obtained from table3  
Population of Male AIDS Patients



The 3D plot in fig 4 shows Aids is endemic for males than females, since AM is exceedingly higher than AF.

The computer output was not programmed to stratify this output according to age, the reasons being that the current research is more concerned about the demographical implication of the disease on the society as well as the financial implication of the disease in the states.

The model suggests that there is steady increase in the population of HIV/AIDS as year progresses (see figs 1 & 2) and the overall budget allocation continue to support less than 1% of the population affected by AIDS/HIV. It, therefore, calls for rapid response to curtail the disease.

The introduction of AIDS/HIV management scheme to popularize the campaign to eradicate the disease, intensify more research on AIDS/HIV on the cost effective drugs to control the diseases is highly desirable. Development cost-effective drugs to control the HIV Virus should be developed. Such drugs should be cheap and affordable by patients with the AIDS/HIV infections.

The government must put in the place legislation on some health care delivery to control the spread of HIV/AIDS through opportunistic sex, unhygienic instruments, blood transfusion etc.

The cost evaluation suggests that a colossal amount of money is needed to support small percentage of the states' population affected with HIV/AIDS.

## **Conclusion**

The computer results indicate that the states will keep on experiencing rapid growth in the occurrence of HIV/AIDS from now to the 2003 and even beyond. The diversifying effect on the life and resources to be lost in the controlling the diseases. This call for urgent actions is taken.

Aids have remained most dreaded monster in the recent times and mankind must begin to eliminate it. So many talented individuals very vibrant manpower are lost due to HIV/AIDS. This disease must therefore be eradicated.

We note that the available data (table 1) was the year 1994 - 1997, our model has allowed us to extrapolate the project cases for year 1998 - 2003. This allows us to have an insight about the future occurrence of the disease and also to put in place strategic policies to avert the spread of the disease.

The computer-simulated result can also be adapted to projected future occurrence the disease above 2003 and cost implication to provide life-supporting drugs for patient affected by the disease.

## **References**

1. Anthony Harries (1989), The Clinical Spectrum of HIV infection in Africa. African Health June/July, 35 - 39.
2. Analyses Des Comportment sexual er France Investigation, 1992, AIDS and sexual behaviour in France, Nature 362, 407 - 409.
3. Anderson, R. M.; May, R. M.; Body, M. C.; Garnet, A. R. and Rowley, J. T. (1991). The spread of HIV-1 in Africa: sexual contact pattern and the demographic impact of AIDS. Nature 352, 581 - 599.
4. Anderson, R. M.; May, R. M. and Rowley, J. T. (1992), Age-dependent choice of

sexual partners and the transmission dynamics of HIV in sub-Saharan Africa. *Phil. Trans. R. Soc. Lond. B.* 336, 135 - 155.

5. Demaris Christener (1999), How HIV attacks Immune System. *The Nigerian Science Guardian* Vol. 15, No. 7, 494 P. 25 - 26.
6. Duns tan F. And Pickles J., "Modeling the spread of AIDS" *Statistics in Medicine. Proc. Conference Application of Statistics in Medicine. Held at University of Wales College. J. D. Griffiths and K. A. Wheeler (editors).*
7. Garnett, G. P. and Anderson, (1993), Factors controlling the spread of HIV in heterosexual communities in developing countries: pattern of mixing between different age and sexual activity classes. *Phil. Trans. R. Soc. Lond. B.* 342, 135 - 159.
8. ... (1993), No reason for complacency about the potential demographic impact of AIDS in Africa. *Trans R Jour. Tropical Med. Hygiene* 87, Suppl. 1, 19 - 22.
9. Geoffrey P. Garnet and Roy, M. Anderson (1994), Balancing sexual partnership in an age and activity stratified model of HIV transmission in heterosexual populations. *IMA Math Appli. Med. And Biology* 11, 161 - 192.
10. Chris Ishiche (1998), *Stopaids International: African's number one Health Journal on Aids.* Vol. No. 4, 1998.
11. Momon, M. A. (1998), Incidence of HIV in Health Care Institution: A problem for Health Care Providers. A paper presented at the National Association of Nigerian Nurses and Midwives Workshop at Abuja, Nigeria.
12. R. I. Kanu (1997), *Medical Consult, a workshop at University of Lagos Teaching Hospital Lagos, Nigeria.*