CLASSICAL FOUNDATIONS: TWO VERSIONS OF MAXWELL'S EQUATIONS

TEPPER L. GILL AND WOODFORD W. ZACHARY

Department of Electrical Engineering, Howard University, Washinton DC USA
e-mail: tgill@howard.edu, wwzachary@earthlink.net

Abstract

In this paper, we provide an introduction to the Howard University research program on the canonical proper-time formulation of classical electrodynamics. The purpose is to develop a consistent classical foundation for relativistic uantum theory. Here, we first show that there are two versions of Maxwell's equations, which are mathematically but not physically equivalent. (Thus, a fundamental conclusion of our research program is that mathematical equivalence is not always related to physical equivalence.) The new version fixes the clock of the field source for all inertial observers (unique definition of simultaneity). However, the (natural definition of the) speed of light is no longer an invariant for all observers, but depends on the motion of the source. This approach allows us to account for radiation reaction without the Lorentz-Dirac equation, self-energy (divergence), advanced potentials or any assumptions about the structure of the source. The theory induces a new invariance group which, in general, is a nonlinear and nonlocal representation of the Lorentz group. The corresponding particle theory is independent of particle number, noninvariant under time reversal (arrow of time), compatible with quantum theory and has an associated positive definite canonical Hamiltonian associated with the clock of the source. Furthermore, as suggested by experiment, a charged particle undergoes (finite) mass renormalization in the presence of a field.

Key words and phrases. special relativity, proper time, radiation reaction.

1. Introduction

In this paper, we provide an outline of the research program at Howard University on the development of a completely physically motivated representation of classical electrodynamics. Our philosophy is based on the following assumptions (or beliefs):

(1) Mathematics provides a set of tools for constructing faithful representations of physical reality but does not dictate the final outcome.
(2) Given the infinite number of possible mathematical tools and structures possible, there is no a prior reason that we cannot construct representation (s) of physical reality that corresponds to the way the world appears to us in our consciousness.
(3) The current (intellectual) state of affairs in physics is not due to past technical mistakes but those of a philosophical and conceptual nature.

The specific goals our program are:

(1) To use and/or develop mathematics that is clearly motivated by physics or clearly stated philosophical (physical) principles.
(2) To carefully study the historical, conceptual and philosophical background to both classical electrodynamics and relativistic quantum theory in order to identify all open problems and/or unanswered questions as clearly discussed by the founding fathers.
(3) To solve the problems or answer the questions of goal two.

1. BACKGROUND (HISTORY)

Einstein begins his 1905 [1], paper with the statement:

> *It is known that Maxwell's electrodynamics as usually understood at the present time - when applied to moving bodies, leads to asymmetries which do not appear to be inherent in the phenomena.*

After quoting a few examples and the unsuccessful attempts of experimenters to discover the light medium (ether), he concludes that mechanics as well as electrodynamics possess no properties corresponding to absolute rest. Thus, the laws of electrodynamics and optics will be valid for all frames in which the equations of mechanics holds. He then suggests that we raise his conjecture to the status of a postulate called the "principle of relativity". He then adds one other postulate to provide what is now known as the basic postulates of the special theory of relativity:

(1) The physical laws of nature and the results of all experiments are independent of the inertial frame of the observer.

(2) The speed of light (relative to all inertial observers) is constant.

Today, after such a long time, many assume that there is ample experimental evidence to support this second assumption. However the very nature of a postulate means that it is a basic assumption of the theory, presented without proof. In fact, if proof were available the postulate would not be needed. Indeed, as Einstein rightly points out in the first footnote to his second paper [4]: "The principle of the constancy of the velocity of light is of course contained in Maxwell's equations." What he meant by this was that the second postulate follows from the fact that the constant c in Maxwell's equations (as currently formulated) is an invariant for all (inertial) observers. Since that time, many experiments have been done to verify this assumption. (Experiments have verified that the speed of light from a source at rest in an inertial frame is constant with value c.) However, in 1965, Fox [41] wrote a very important paper which reviewed the evidence for constant c and against the emission theory of Ritz [8]. His conclusion was that all previous experiments were flawed for a number of reasons. In many cases, analysis of the experimental data failed to take into account the (now well-known) extinction theorem of Ewald and Oseen (see Jackson [42]). The only data found that firmly supported the second postulate came from experiments on the lifetime of fast mesons and the velocity of rays and light from moving sources. In his conclusion Fox states that:

> *Unless something has been overlooked, these seem to be the only pieces of experimental evidence we have. This is surprising in light of the long history and importance of the problem.*

(We will return to this later and show that the experiments on the lifetime of fast mesons and the velocity of rays and light from moving sources are inconclusive.)

As noted by Bridgman [2], the special theory allows us to by-pass but not answer the fundamental question of "the nature of the physical mechanism by which objects are lighted". From an operational point of view we must ask if it is physically possible to consider light as a "thing" that travels? Bridgman [6] observed that:

> *We can give no operational meaning to the idea that light exists at each point between source and sink. The idea of light as a thing traveling is pure invention based on sense perceptions and the mechanical world view.*

Bridgman further points out that the special theory of relativity spreads time over space by assuming light is a thing traveling. Hence if we assume that light is the transfer of energy, conservation of energy requires that we integrate the local energy density over all space at a definite time instant, which puts us in a logical circle, as this implies the non-local nature of light.

As noted in Miller [3], Einstein chose to consider light as a thing traveling for convenience. This allowed him to use the standard notion of velocity for measurement purposes. However, in the special theory, light is not a material particle nor is it a wave, since if it's a particle its velocity cannot be independent of the source motion and, if it's a wave, it must travel in a medium (the ether), which is known to not have any effect on light!

It should not go unnoticed that, in a paper published almost at the same time (a few months earlier), Einstein [9] used the the concept of light as a "localized enegy packet" to explain the photoelectric effect. In fact, Planck [5] wrote:

> *According to the latest statements by Einstein it would be necessary to assume that free radiation in vacuum, and hence light waves themselves, has an atomistic constitution, and thus to abandon Maxwell's equations.*

We should not be amazed at Planck's statement since, at the time, the question of the need for Maxwell's equations at all was still an open subject. In 1867, Ludvig Lorentz [43] introduced the retarded vector and scalar potentials. It was shown that these led to the same results obtained by Maxwell via the introduction of the displacement current into the Amp`ere's law. Indeed, it has been known since then that all the results of the

Maxwell theory can be obtained directly from the potentials, without ever introducing fields. (It has recently been shown by Hamdan, Hariri and Lopez-Bonilla [7] that one can derive Maxwell's equations directly from the Lorenz force.)

There were many who took L. Lorentz's position, but the major protagonist in this debate was Walther Ritz [8]. Ritz, like Einstein, accepted H. A. Lorentz's theory of the electron but rejected the ether. He further noted that, from a strictly logical point of view, Maxwell's electric and magnetic fields, which appear to play such an important role can be entirely eliminated from the theory. He argued that, in reality, Maxwell's theory deals only with certain relations between space and time. In his view, we could simply return to the elementary actions via retarded potentials (now known as action-at-a-distance). He further pointed out that the field equations had an infinite number of solutions that are incompatible with experiment and, in order to eliminate these extraneous solutions, it is necessary to adopt the retarded potentials anyway. This introduces an additional assumption which is not needed if we start with the retarded potentials in the first place. Einstein did not completely accept, but was swayed by Ritz's position. Indeed in his 1909 paper [9], Einstein stated:

> *According to the usual theory, an oscillating ion generates a divergent spherical wave. The reverse process does not exist as a elementary process. The convergent wave is indeed mathematically possible; but for its approximate realisation an enormous number of elementary emitting elementary systems would be required. Hence the elementary process of light-emission has not as such the character of reversibility. Herein, I believe, our wave theory is incorrect. It seems in relation to this point Newton's emission theory contains more truth then the wave theory, for the energy communicated to a light-particle in emission is not spread over infinite space but remains available for an elementary process of absorption.*

Here, Einstein is agreeing with Ritz's position that retarded potentials express the elementary process of emission, whereas Maxwell's equations do not. We get a further clue to Einstein's thinking on this subject from his Autobiographical notes of 1949 [10] (see Brown [11]).

> *Reflections of this type made it clear to me as long ago as 1900, i.e., shortly after Planck's trailblazing work, that neither mechanics nor lectrodynamics could (except in limiting cases) claim exact validity.*

Brown points out that, because he was not sure that Maxwell's theory would survive the existence of photons, Einstein derived the Lorentz transformations from kinematical arguments, as opposed to the symmetry properties of Maxwell's equations. He believed that the Lorentz transformations were fundamental and would survive any failures in the Maxwell theory.

In the past, there always was a certain tension between field theory and action-at-a-distance. The most famous recent work on the subject is the Wheeler-Feynman formulation of classical electrodynamics [12], in which they eliminate the field completely (in favor of an action-at-a-distance approach) in order to solve the self-energy divergence problem associated with the then accepted Dirac theory [13]. However, among other things, the need for both advanced and retarded interactions, the inability to quantize and the intrinsic usefulness of the self-energy divergence for the success of quantum electrodynamics became important reasons for its lack of favor as a replacement for the Dirac approach.

Purpose: In this paper, we introduce the canonical proper-time formulation of classical electrodynamics, where the local clock of the moving system replaces the clock of the observer. This approach is mathematically equivalent but is not physically equivalent to the conventional approach. Physically, this change is equivalent to a new definition of velocity for relativistic systems and by-passes all the wellknown problems with the conventional theory. We also develop the corresponding particle theory, which produces a positive definite canonical Hamiltonian and, the local clock is non-invariant under time-reversal (time-arrow) The main purpose of our efforts is to clear up the classical problems in order to prepare the way for a consistent relativistic quantum theory.

2. Canonical Proper-time Classical Theory

2.1. Observers and Observed Systems.

In actual experimental setups there is an observer and a system to be observed. The observer has his/her own inertial frame of reference, including clocks and measuring equipment.) There are few (if any) experiments of interest conducted on systems with constant velocity. In general, some interaction is required, so that the system responds to forces. After sufficient data is obtained and analyzed (based on current theoretical guildelines) a report of the findings is prepared. These are the essentials of the process. The first postulate of the special theory of relativity imposes a natural constraint on the extent that we may believe in the results of the experiment; namely, that any other observer, using similar equipment in any other inertial frame of reference must be able to obtain results that differ, at most, by a Lorentz transformation.

It was natural for Einstein to use the clock of the observer to measure time. The recognition that this constraint on theory is a convention is a major thesis of our research program. We will show that an equally valid clock to use is the clock of the observed system, which is generally known as the proper-time. (In this terminology, the conventional clock used is the proper-time of the observer.)

2.2. Maxwell Theory.

In order to formulate the local-time version of Maxwell's equations, it is convenient to start with the standard definition of proper-time:

$$d\tau^2 = dt^2 - \frac{1}{c^2}d\mathbf{x}^2 = dt^2\left[1 - \frac{\mathbf{w}^2}{c^2}\right], \quad \mathbf{w} = \frac{d\mathbf{x}}{dt}.$$

Motivated by geometry and (mathematical) philosophy, Minkowski introduced the concept of proper time (first discovered by Poincar´e). Recently, it has been suggested by Damour [14] that Minkowski was not aware that $d\tau$ is not an exact one-form and hence cannot be used for a metric. Thus, he did not completely understand its physical meaning, since a major conclusion of Einstein was that a moving system measures time differently compared to one at rest. (For very interesting additional discussion on this and other related points, see Walter [15] and included references.) It is clear that Minkowski became aware of this fact eventually, if Sommerfeld is to be believed (see his notes in [16] after the translation of Minkowski's paper (pg. 94)). Nevertheless, some of the mathematically inclined have dismissed this (physical) fact by attaching a "co-moving observer" on the tangent curve (bundle) of the moving particle in order to induce an instantaneous exact one-form for the fourgeometry at each time slice. (This is mathematically correct but physical nonsense.) However, there is an important physical reason why $d\tau$ is not an exact (mathematical) one-form. Physically, a particle can traverse many different paths (in space) during any given τ interval. This reflects the fact that the distance traveled in a given time interval depends on the forces acting on the particle. This suggests that the actual clock of the source carries additional physical information, and there is no a priori (physical) reason to believe that this information is properly encoded in the clock of a mathematical co-moving observer. In order to see that this is the case, rewrite the above equation as:

$$dt^2 = d\tau^2 + \frac{1}{c^2}d\mathbf{x}^2 = d\tau^2\left[1 + \frac{\mathbf{u}^2}{c^2}\right], \quad \mathbf{u} = \frac{d\mathbf{x}}{d\tau}.$$

For any other observer, we have:

$$dt'^2 = d\tau^2 + \frac{1}{c^2}d\mathbf{x}'^2 = d\tau^2\left[1 + \frac{\mathbf{u}'^2}{c^2}\right], \quad \mathbf{u}' = \frac{d\mathbf{x}'}{d\tau}.$$

Thus, all observers can use one unique clock to discuss all events associated with the source (simultaneity). Returning to the first equation, we see that the new metric defined by dt is clearly exact, while the representation space is now Euclidean. However, the natural definition of velocity is no longer $\mathbf{w} = d\mathbf{x}/dt$ but $\mathbf{u} = d\mathbf{x}/d\tau$.

This fact suggests that there may be a certain duality in the relationship between $t,\ \tau$ and w, u. To see that this is indeed the case, recall that

$\mathbf{u} = \mathbf{w}\Big/\sqrt{1-(\mathbf{w}^2/c^2)}.$ Solving for w, we get that $\mathbf{w} = \mathbf{u}\Big/\sqrt{1+(\mathbf{u}^2/c^2)}.$

If we set $b = \sqrt{c^2+\mathbf{u}^2}$, this relationship can be written as

$$\text{(1)} \qquad \frac{\mathbf{w}}{c} = \frac{\mathbf{u}}{b}.$$

For reasons to be clear momentarily, we call b the collaborative speed of light.
Indeed, we see that

$$\text{(2)} \qquad \frac{1}{c}\frac{\partial}{\partial t} = \frac{1}{c}\frac{\partial \tau}{\partial t}\frac{\partial}{\partial \tau} = \frac{1}{c}\frac{1}{\sqrt{1+(\mathbf{u}^2/c^2)}}\frac{\partial}{\partial \tau} = \frac{1}{b}\frac{\partial}{\partial \tau}.$$

For any other observer, it is easy to see that the corresponding result will be:

$$\text{(3)} \qquad \frac{\mathbf{w}'}{c} = \frac{\mathbf{u}'}{b'}, \qquad \frac{1}{c}\frac{\partial}{\partial t'} = \frac{1}{b'}\frac{\partial}{\partial \tau}.$$

We see from equation (2) and the last part of equation (3) that the invariance of *c* on the left is replaced by invariance of τ on the right. (These equations clearly represent mathematically equivalent relations.) In order to see their impact on Maxwell's equations, write them (in c.g.s. units)

$$\text{(4)} \qquad \begin{aligned} &\nabla\cdot\mathbf{B} = 0, \qquad \nabla\cdot\mathbf{E} = 4\pi\rho, \\ &\nabla\times\mathbf{E} = -\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t}, \quad \nabla\times\mathbf{B} = \frac{1}{c}\left[\frac{\partial\mathbf{E}}{\partial t} + 4\pi\rho\mathbf{w}\right]. \end{aligned}$$

Using equations (1) and (2) in (4), we have (the identical mathematical representation for Maxwell's equations):

$$\text{(5)} \qquad \begin{aligned} &\nabla\cdot\mathbf{B} = 0, \qquad \nabla\cdot\mathbf{E} = 4\pi\rho, \\ &\nabla\times\mathbf{E} = -\frac{1}{b}\frac{\partial\mathbf{B}}{\partial \tau}, \quad \nabla\times\mathbf{B} = \frac{1}{b}\left[\frac{\partial\mathbf{E}}{\partial \tau} + 4\pi\rho\mathbf{u}\right]. \end{aligned}$$

Thus, we see that Maxwell's equations are equally valid when the local time of the particle is used to describe the fields. This leads to the following conclusions:

1) There are two distinct clocks to use in the representation of Maxwell's equations. (Thus, the choice of clocks is a convention in the true sense of Poincare.)
2) Since the two representations are mathematically equivalent, we conclude that mathematical equivalence is not physical equivalence. (This will be clear after we derive the corresponding wave equation below.)

3) When the local clock of the system is used, the constant speed of light c is replaced by the collaborative speed of light b, which depends on the motion of the system (e.g., $b = \sqrt{c^2 + \mathbf{u}^2}$).
4) There is another group (closely related to the Lorentz group) which fixes the local-time of the particle for all observers. Before constructing the group, we derive the corresponding wave equations in the local-time variable. Taking the curl of the last two equations in (5), and using standard vector identities, we get:

$$
\begin{aligned}
&\frac{1}{b^2}\frac{\partial^2 \mathbf{B}}{\partial \tau^2} - \frac{\mathbf{u}\cdot\mathbf{a}}{b^4}\left[\frac{\partial \mathbf{B}}{\partial \tau}\right] - \nabla^2\cdot\mathbf{B} = \frac{1}{b}\left[4\pi\nabla\times(\rho\mathbf{u})\right], \\
(6)\qquad &\frac{1}{b^2}\frac{\partial^2 \mathbf{E}}{\partial \tau^2} - \frac{\mathbf{u}\cdot\mathbf{a}}{b^4}\left[\frac{\partial \mathbf{E}}{\partial \tau}\right] - \nabla^2\cdot\mathbf{E} = -\nabla(4\pi\rho) - \frac{1}{b}\frac{\partial}{\partial \tau}\left[\frac{4\pi(\rho\mathbf{u})}{b}\right],
\end{aligned}
$$

Where $\mathbf{a} = d\mathbf{u}/d\tau$ is the collaborative acceleration caused by external forces. Thus, we see that a new term arises when the proper-time of the system is used to describe the fields. This makes it clear that the local clock encodes information about the particle's interaction that is unavailable when the clock of the observer is used to describe the fields, and shows clearly that physical equivalence is not the same as mathematical equivalence. The new term in equation (6) is dissipative, acts to oppose the acceleration, is zero when a = 0 and, arises instantaneously with the action of forces on the particle. Furthermore, as expected, this term does not depend on the nature of the force causing the acceleration. This is exactly what one expects of the back reaction caused by the inertial resistance of the particle to accelerated motion and, according to Wheeler and Feynman [12], is precisely what is meant by radiation reaction. Thus, the collaborative use of the observer's coordinate system and the local clock of the observed system provides intrinsic information about the field dynamics not available in the conventional formulation of Maxwell's theory If we make a scale transformation (at fixed position) with $\mathbf{E} \rightarrow (b/c)^{1/2}\mathbf{E}$ and $\mathbf{B} \rightarrow (b/c)^{1/2}\mathbf{B}$, the equations in (6) transform to

$$
\begin{aligned}
&\frac{1}{b^2}\frac{\partial^2 \mathbf{B}}{\partial \tau^2} - \nabla^2\cdot\mathbf{B} + \left[\frac{\ddot{b}}{2b^3} - \frac{3\dot{b}^2}{4b^4}\right]\mathbf{B} = \frac{c^{1/2}}{b^{3/2}}\left[4\pi\nabla\times(\rho\mathbf{u})\right], \\
(7)\qquad &\frac{1}{b^2}\frac{\partial^2 \mathbf{E}}{\partial \tau^2} - \nabla^2\cdot\mathbf{E} + \left[\frac{\ddot{b}}{2b^3} - \frac{3\dot{b}^2}{4b^4}\right]\mathbf{E} = -\frac{c^{1/2}}{b^{1/2}}\nabla(4\pi\rho) - \frac{c^{1/2}}{b^{3/2}}\frac{\partial}{\partial \tau}\left[\frac{4\pi(\rho\mathbf{u})}{b}\right].
\end{aligned}
$$

This is the Klein-Gordon equation with an effective mass μ given by

$$\mu = \left\{ \frac{\hbar^2}{c^2} \left[\frac{\ddot{b}}{2b^3} - \frac{3\dot{b}^2}{4b^4} \right] \right\}^{1/2} = \left\{ \frac{\hbar^2}{c^2} \left[\frac{\mathbf{u} \cdot \ddot{\mathbf{u}} + \dot{\mathbf{u}}^2}{2b^4} - \frac{5 \left(\mathbf{u} \cdot \dot{\mathbf{u}} \right)^2}{4b^6} \right] \right\}^{1/2} .$$

.

Thus, the new dissipative term is equivalent to an effective mass that arises due to the collaborative acceleration of the particle. This means that the cause for radiation reaction comes directly from the use of the local clock to formulate Maxwell's equations. Thus, in this approach, there is no need to assume advanced potentials, self-interaction, mass renormalization along with the Lorentz-Dirac equation in order to account for it (radiation reaction), as has been required when the observer clock is used. Furthermore, no assumptions about the structure of the charge are required.

2.3. Proper-time Group. We now identify the new transformation group that preserves the first postulate of the special theory. The standard (Lorentz) time transformations between two inertial observers can be written as

$$(8) \qquad t' = \gamma(\mathbf{v}) \left[t - \mathbf{x} \cdot \mathbf{v}/c^2 \right], \qquad t = \gamma(\mathbf{v}) \left[t' + \mathbf{x}' \cdot \mathbf{v}/c^2 \right].$$

We want to replace $t,\ t'$ by τ. To do this, use the relationship between dt and $d\tau$ to get:

$$(9) \qquad t = \tfrac{1}{c} \int_0^\tau b(s) ds = \tfrac{1}{c} \bar{b} \tau, \quad t' = \tfrac{1}{c} \int_0^\tau b'(s) ds = \tfrac{1}{c} \bar{b}' \tau,$$

Where we have used the mean value theorem of calculus to obtain the end result, so that both $\bar{b}$ and $\bar{b}'$ represent an earlier τ-value of b and b' respectively. Note that, as b and b' depend on τ, the transformations (9) represent explicit nonlinear relationships between $t,\ t'$ and τ (during interaction). (This is to be expected in the general case when the system is acted on by external forces.) If we set

$$\mathbf{d}^* = \mathbf{d}/\gamma(\mathbf{v}) - (1 - \gamma(\mathbf{v})) \left[(\mathbf{v} \cdot \mathbf{d})/(\gamma(\mathbf{v})\mathbf{v}^2) \right] \mathbf{v},$$

we can write the transformations that fix τ as:

$$\mathbf{x}' = \gamma(\mathbf{v}) \left[\mathbf{x}^* - (\mathbf{v}/c)\bar{b}\tau \right], \qquad \mathbf{x} = \gamma(\mathbf{v}) \left[\mathbf{x}'^* + (\mathbf{v}/c)\bar{b}'\tau \right],$$

$$(10) \quad \mathbf{u}' = \gamma(\mathbf{v}) \left[\mathbf{u}^* - (\mathbf{v}/c)b \right], \qquad \mathbf{u} = \gamma(\mathbf{v}) \left[\mathbf{u}'^* + (\mathbf{v}/c)b' \right],$$

$$\mathbf{a}' = \gamma(\mathbf{v}) \left\{ \mathbf{a}^* - \mathbf{v} \left[(\mathbf{u} \cdot \mathbf{a})/(bc) \right] \right\}, \quad \mathbf{a} = \gamma(\mathbf{v}) \left\{ \mathbf{a}'^* + \mathbf{v} \left[(\mathbf{u}' \cdot \mathbf{a}')/(b'c) \right] \right\}.$$

If we put equation (9) in (8), differentiate with respect to τ and cancel the extra factor of c, we get the transformations between $b \text{ and } b'$:

$$(11)\ b'(\tau) = \gamma(\mathbf{v})\left[b(\tau) - \mathbf{u}\cdot\mathbf{v}/c\right], \qquad b(\tau) = \gamma(\mathbf{v})\left[b'(\tau) + \mathbf{u}'\cdot\mathbf{v}/c\right].$$

Equations (10) in (11) provide an explicit nonlinear representation of the Lorentz group, which uses the local clock to describe the dynamics of the system and preserves the first postulate of the special theory (the only one that really matters). It was shown in [17] that Maxwell's equations transform in the same way as in the conventional theory. However, the charge and current density have the following transformations:

$$(12) \qquad \mathbf{J}' = \mathbf{J} + (\gamma - 1)\frac{(\mathbf{J}\cdot\mathbf{v})}{\mathbf{v}^2}v - \gamma\frac{b}{c}\rho\mathbf{v},$$

$$(13) \qquad b'\rho' = \gamma(\mathbf{v})\left[b\rho - (\mathbf{J}\cdot\mathbf{v}/c)\right].$$

Using the first equation of (11) in (13), we get:

$$(14) \qquad \rho' = \frac{\rho - (\mathbf{J}\cdot\mathbf{v}/bc)}{1 - (\mathbf{u}\cdot\mathbf{v}/bc)}.$$

This result is different from the standard one, (which we obtain if we set $b' = b = c$ in (13))

$$\rho' = \gamma(\mathbf{v})\left[\rho - (\mathbf{J}\cdot\mathbf{v}/c^2)\right].$$

Furthermore, if we insert the expression $\mathbf{J}/c = \rho(\mathbf{u}/b)$ in (14); we obtain

$$(15) \qquad \rho' = \rho\frac{1 - (\mathbf{u}\cdot\mathbf{v}/b^2)}{1 - (\mathbf{u}\cdot\mathbf{v}/bc)}.$$

2.4. Canonical Proper-Time Particle Theory.

Since we desire complete compatibility with quantum theory, it is natural to require that any change from the observer clock to the local clock of the observed system be a canonical change. The key concept to our approach may be seen by examining the time evolution of a dynamical parameter W(x, p), via the standard formulation of classical mechanics, described in terms of the Poisson brackets:

$$(16) \qquad \frac{dW}{dt} = \{H, W\} + \frac{\partial W}{\partial t}.$$

We can also represent the dynamics using the proper (or local) time of the system by using the representation $d\tau = (1/\gamma)dt = (mc^2/H)dt$, so that:

$$\frac{dW}{d\tau} = \frac{dt}{d\tau}\frac{dW}{dt} = \frac{H}{mc^2}\{H, W\} + \frac{\partial W}{\partial \tau}.$$

Assuming a well-defined (invariant) rest energy (mc^2) for the system, we determine the canonical proper-time Hamiltonian K such that:

$$\{K, W\} = \frac{H}{mc^2}\{H, W\}, \quad K|_{\mathbf{p}=0} = H|_{\mathbf{p}=0} = mc^2.$$

Using

$$\begin{aligned}\{K, W\} &= \left[\frac{H}{mc^2}\frac{\partial H}{\partial \mathbf{p}}\right]\frac{\partial W}{\partial \mathbf{x}} - \left[\frac{H}{mc^2}\frac{\partial H}{\partial \mathbf{x}}\right]\frac{\partial W}{\partial \mathbf{p}} \\ &= \frac{\partial}{\partial \mathbf{p}}\left[\frac{H^2}{2mc^2} + a\right]\frac{\partial W}{\partial \mathbf{x}} - \frac{\partial}{\partial \mathbf{x}}\left[\frac{H^2}{2mc^2} + a'\right]\frac{\partial W}{\partial \mathbf{p}},\end{aligned}$$

we get that $a = a' = \frac{1}{2}mc^2,$ so that (assuming no explicit time dependence)

$$K = \frac{H^2}{2mc^2} + \frac{mc^2}{2}, \quad \text{and} \quad \frac{dW}{d\tau} = \{K, W\}.$$

Since τ is invariant during interaction (minimal coupling), we make the natural assumption that (the form of) K also remains invariant. Thus, if $\sqrt{c^2\mathbf{p}^2 + m^2c^4} \to \sqrt{c^2\boldsymbol{\pi}^2 + m^2c^4} + V$. The becomes

$$K = \frac{\boldsymbol{\pi}^2}{2m} + mc^2 + \frac{V^2}{2mc^2} + \frac{V\sqrt{c^2\boldsymbol{\pi}^2 + m^2c^4}}{mc^2}.$$

If we set $H_0 = \sqrt{c^2\boldsymbol{\pi}^2 + m^2c^4},$ use standard vector identities with $H_0 = mcb$ and $\nabla \times \boldsymbol{\pi} = -\frac{e}{c}\mathbf{B}$ and, compute Hamilton's equations, we get:

$$\mathbf{u} = \frac{d\mathbf{x}}{d\tau} = \left[1 + \frac{V}{H_0}\right]\frac{\boldsymbol{\pi}}{m} = \frac{\boldsymbol{\pi}}{\tilde{m}},$$

$$\begin{aligned}(17) \qquad \frac{d\mathbf{p}}{d\tau} &= -\frac{\left[(\boldsymbol{\pi}\cdot\nabla)\boldsymbol{\pi} + \frac{e}{c}\boldsymbol{\pi}\times\mathbf{B}\right]}{m}\left[1 + \frac{V}{H_0}\right] - \nabla V\frac{H_0}{mc^2}\left[1 + \frac{V}{H_0}\right] \\ &= \frac{e}{c}(\mathbf{u}\cdot\nabla)\mathbf{A} + \frac{e}{c}\mathbf{u}\times\mathbf{B} - \nabla V\frac{b}{c}\left[1 + \frac{V}{H_0}\right].\end{aligned}$$

We remark that, one can view $\tilde{m}$ as a (finite) renomalization of m, which occurs the moment that the potential is turned on. This may seem strange to one not familiar with the history of this subject. An excellent discussion of renomalization from a historical perspective can be found in the article by Dresden [18]. In order to see the impact of our condition that K remains invariant during interaction in another way, compute the Lagrangian from $Ld\tau = \mathbf{p}\cdot d\mathbf{x} - K\,d\tau,$ to get:

$$L = \tilde{m}\mathbf{u}^2 - \frac{\tilde{m}\mathbf{u}^2}{2}\left(\frac{\tilde{m}}{m}\right) - mc^2 - \frac{V^2}{2mc^2} - V\left(\frac{b}{c}\right) + \frac{e}{c}\mathbf{A}\cdot\mathbf{u}.$$

However, if we use the fact that $\pi = \tilde{m}\mathbf{u}$ directly in H_0, we get the implicit relation $b = \sqrt{c^2 + \frac{\tilde{m}^2\mathbf{u}^2}{m^2}}$. If we use this in our equation for the metric, we get (in spherical coordinates):

$$dt^2 = \left[1 + \frac{\mathbf{u}^2}{c^2\left[1 + \frac{V}{H_0^2}\right]^2}\right] d\tau^2 \Rightarrow$$

$$c^2dt^2 = c^2d\tau^2 + \frac{d\mathrm{x}^2}{\left[1 + \frac{V}{H_0}\right]^2} = c^2d\tau^2 + \frac{dr^2}{\left[1 + \frac{V}{H_0}\right]^2} + \frac{r^2d\theta^2}{\left[1 + \frac{V}{H_0^2}\right]^2} + \frac{r^2\sin^2\theta d\phi^2}{\left[1 + \frac{V}{H_0}\right]^2}.$$

Thus, we see that the metric becomes deformed in the presence of a potential (e.g., geometry is created by physics). If we multiply the equation (17) by b/c, compute the material derivative of A with respect to τ and use the definition of the E, with $V = e\Phi$, we have:

$$\frac{c}{b}\left[\frac{d\mathbf{p}}{d\tau} - \frac{e}{c}\frac{d\mathbf{A}}{d\tau}\right] = -\frac{e}{b}\frac{\partial\mathbf{A}}{\partial\tau} + \tfrac{e}{b}\mathbf{u}\times\mathbf{B} - e\nabla\Phi\left[1 + \frac{V}{mcb}\right] \tag{18}$$
$$= e\mathbf{E} + \tfrac{e}{b}\mathbf{u}\times\mathbf{B} - e\nabla\Phi\frac{V}{mcb}.$$

It has been observed by Feynman [19] that, although there is experimental evidence for the existence of electromagnetic mass, the conventional theory "falls on its face" in accounting for this mass "..., because it does not produce a consistent theory–and the same is true for quantum modifications". The last term in equation (18) is an addition to the Lorentz force, with the opposite sign of $-\nabla\Phi$, which appears in E. In order to see the physical meaning of the term, assume an interaction between a proton and an electron, with A = 0, so that (18) becomes:

$$\mathbf{u} = \left[1 + \frac{V}{mc^2}\right]\frac{\pi}{m},$$

$$\frac{c}{b}\frac{d\mathbf{p}}{d\tau} = -\nabla V - \nabla V\frac{V}{mcb}.$$

If we treat u as (approximately) $\mathbf{p}/m$ and set b = c, we get that

$$m\mathbf{a} = -\nabla V - \nabla V\frac{V}{mc^2}.$$

In this case, its easy to show that the classical electron radius, r0 is a critical point $(\text{e.g.}, -\nabla\Phi - \nabla\Phi(V/mc^2) = 0)$. Thus, for $0 < r < r_0$, the force becomes repulsive. We interpret this as a fixed region of repulsion, so that singularity r = 0 is impossible to reach at the classical level. The neglected terms are attractive but of lower order. This makes the critical point slightly less than r0. Thus, in general, the electron experiences a strongly repulsive force when it gets too close to the proton. This means that the principle of impenetrability, namely that no two particles can occupy the same space at the same time, is upheld. Furthermore, this analysis shows conclusively that information about the structure of the particle is not required in the canonical proper-time theory. Finally, it is clear that the neglect of second order terms, give us the nonrelativistic theory.

3. Many-Particle Case

Once it was agreed that the proper Newtonian theory should be invariant under the Lorentz group, work on this problem was generally ignored until after World War Two when it was realized that quantum theory did not solve the open problems of classical electrodynamics. In particular, it was first noticed that the canonical center-of-mass is not the three-vector part of a four-vector (see Pryce [20]). The well-known no-interaction theorem shows that it is impossible to construct a (interacting) relativistic many-particle theory that allows covariance and independent particle world-lines. Thus, the four-vector approach "falls on its face" for more than one particle. (For a discussion of this and all known problems, see [21]). In this section we construct a consistent classical (relativistic) many-particle theory that is quantizable and includes Newtonian mechanics.

Suppose we have a closed system of *n* interacting particles with Hamiltonians

Hi and total Hamiltonian H. We assume that H is of the form $H = \sum_{i=1}^{n} H_i$. If we define the effective mass M and total momentum P by

$$Mc^2 = \sqrt{H^2 - c^2\mathbf{P}^2}, \quad \mathbf{P} = \sum_{i=1}^{n} \mathbf{p}_i,$$

then H also has the representation $H = \sqrt{c^2\mathbf{P}^2 + M^2c^4}$. To construct the manyparticle theory, we observe that the representation $d\tau = (Mc^2/H)dt$ does not depend on the number of particles in the system and is an invariant for all observers. Thus, we can uniquely define the proper-time of the system for all observers. If we let L be the boost (generator of pure Lorentz transformations) and define the total angular momentum J by

$$\mathbf{J} = \sum_{i=1}^{n} \mathbf{x}_i \times \mathbf{p}_i,$$

,

we then have the following Poisson Bracket relations characteristic of the Lie algebra for the Poincar´e group (when we use the observer proper-time):

$$\frac{d\mathbf{P}}{dt} = \{H, \mathbf{P}\} = \mathbf{0} \qquad \frac{d\mathbf{J}}{dt} = \{H, \mathbf{J}\} = \mathbf{0} \qquad \{P_i, P_j\} = 0$$

$$\{J_i, P_j\} = \varepsilon_{ijk}P_k \qquad \{J_i, J_j\} = \varepsilon_{ijk}J_k \qquad \{J_i, L_j\} = \varepsilon_{ijk}L_k$$

$$\frac{d\mathbf{L}}{dt} = \{H, \mathbf{L}\} = -\mathbf{P} \qquad \{P_i, L_j\} = -\delta_{ij}H/c^2, \qquad \{L_i, L_j\} = -\varepsilon_{ijk}J_k/c^2.$$

It is easy to see that M commutes with H, P, and J, and to show that M commutes with L. Constructing K as in the one-particle case, we have

$$K = \frac{H^2}{2Mc^2} + \frac{Mc^2}{2} = \frac{\mathbf{P}^2}{2M} + Mc^2.$$

Thus, we can use the same definitions for P, J, and L to obtain our new commutation relations:

$$\frac{d\mathbf{P}}{d\tau} = \{K, \mathbf{P}\} = \mathbf{0}, \quad \frac{d\mathbf{J}}{d\tau} = \{K, \mathbf{J}\} = \mathbf{0}, \quad \{P_i, P_j\} = 0,$$

$$\{J_i, P_j\} = \varepsilon_{ijk}P_k, \quad \{J_i, J_j\} = \varepsilon_{ijk}J_k, \quad \{J_i, L_j\} = \varepsilon_{ijk}L_k,$$

$$\frac{d\mathbf{L}}{d\tau} = \{K, \mathbf{L}\} = \frac{-H}{Mc^2}\mathbf{P}, \quad \{P_i, L_j\} = -\delta_{ij}H/c^2, \quad \{L_i, L_j\} = -\varepsilon_{ijk}J_k/c^2.$$

It follows that, except for a constant scale change, the inhomogeneous proper-time group is generated by the same Lie algebra as the Poincar´e group. This result is not surprising given the close relation between the two groups. It also proves our earlier statement that the form of K is fully relativistic.

Let the map from $(\mathbf{x}_i, t) \rightarrow (\mathbf{x}_i, \tau)$ be denoted by $\mathbf{C}[t, \tau]$, and let $\mathbf{P}(O', O)$ be the Poincare map from $O \rightarrow O'$.

Theorem 1.

The proper-time coordinates of the system as seen by an observer at O are related to those of an observer at O0 by the transformation:

$\mathbf{R}_M[\tau] = \mathbf{C}[t', \tau]\mathbf{P}(O', O)\mathbf{C}^{-1}[t, \tau].$

Proof. The proof follows since the diagram below is commutative.

$$\begin{array}{ccc} O(\{\mathbf{x}_i\}, t) & \longrightarrow & O'(\{\mathbf{x}'_i\}, t') \\ \mathbf{C}^{-1}[t, \tau] \Big\uparrow & & \Big\downarrow \mathbf{C}[t', \tau] \\ O(\{\mathbf{x}_i\}, \tau) & \longleftarrow & O'(\{\mathbf{x}'_i\}, \tau) \end{array}$$

The top diagram is the Poincar´e map from $O \to O'$. It is important to note that this map is between the coordinates of observers. In this sense, our approach may be viewed as a direct generalization of the conventional theory. In the global case, when U is constant, t is related to τ by a scale transformation so that we have a group with the same Lie algebra as the Poincar´e group (up to a constant scale), but it has an Euclidean metric. In this case, Theorem 1 proves that R_M is in the proper-time group formed by a similarity action on the Poincar´e group by the canonical group $\mathbf{C}_\tau$. On the other hand, Theorem 1 is true in general. This means that in both the local and global cases (when the acceleration is nonzero) t is related to τ_i and τ via nonlocal (nonlinear) transformations. It follows that, in general, the group action is not linear, and hence is not covered by the Cartan classification.

Since K does not depend on the center-of-mass position X, it is easy to see that

$$\mathbf{U} = \frac{d\mathbf{X}}{d\tau} = \frac{\partial K}{\partial \mathbf{P}} = \frac{\mathbf{P}}{M} = \frac{1}{M}\sum_{i=1}^{n} m_i \mathbf{u}_i, \tag{19}$$

where $\mathbf{u}_i = d\mathbf{x}_i/d\tau_i$. We can now define b by

$b = \sqrt{\mathbf{U}^2 + c^2} \Rightarrow H = Mcb.$

Thus, we can represent the relationship between $d\tau$ and dt as:

$d\tau = (c/b)dt.$

If we set $\mathbf{u}_i = \frac{d\mathbf{x}_i}{d\tau_i} = \frac{d\tau}{d\tau_i}\frac{d\mathbf{x}_i}{d\tau} = \frac{b_i}{b}\mathbf{v}_i \Rightarrow \frac{\mathbf{u}_i}{b_i} = \frac{\mathbf{v}_i}{b}.$, an easy calculation shows that $\mathbf{v}_i = d\mathbf{x}_i/d\tau$,.The velocity v_i is the one our observer sees when he uses the global proper-clock of the system to compute the particle velocity, while ui is the one seen when he uses the local proper clock of the particle to compute its velocity. Solving for u_i and b_i in terms of v_i and b, we get

$$\mathbf{u}_i = \frac{c\mathbf{v}_i}{\sqrt{b^2 - \mathbf{v}_i^2}}, b_i = \frac{cb}{\sqrt{b^2 - \mathbf{v}_i^2}} \text{ or } \frac{b_i}{b} = \frac{c}{\sqrt{b^2 - \mathbf{v}_i^2}}.$$

Note that, since $b^2 = \mathbf{U}^2 + c^2,$ if U is not zero, then any v_i can be larger than c. On the other hand, if U is zero, b = c and, from the global perspective, our theory looks like the conventional one. Using (19), we can rewrite U as

$$\mathbf{U} = \frac{1}{M}\sum_{i=1}^{n} m_i \mathbf{u}_i = \frac{1}{M}\sum_{i=1}^{n} \frac{m_i c\mathbf{v}_i}{\sqrt{b^2 - \mathbf{v}_i^2}} = \frac{1}{M}\sum_{i=1}^{n} \frac{b_i m_i \mathbf{v}_i}{b} = \frac{1}{H}\sum_{i=1}^{n} H_i \mathbf{v}_i$$

It follows that the position of the center-of-mass (energy) satisfies

$$\mathbf{U} = \frac{1}{M}\sum_{i=1}^{n} m_i \mathbf{u}_i = \frac{1}{M}\sum_{i=1}^{n} \frac{m_i c\mathbf{v}_i}{\sqrt{b^2 - \mathbf{v}_i^2}} = \frac{1}{M}\sum_{i=1}^{n} \frac{b_i m_i \mathbf{v}_i}{b} = \frac{1}{H}\sum_{i=1}^{n} H_i \mathbf{v}_i.$$

It is natural to choose Y so that X is the canonical center of mass:

$$\mathbf{X} = \frac{1}{H}\sum_{i=1}^{n} H_i \mathbf{x}_i + \frac{c^2(\mathbf{S} \times \mathbf{P})}{H(Mc^2 + H)},$$

where S is the (conserved) spin of the system. The important point is that $(\mathbf{X}, \mathbf{P}, \tau, K)$ is the new set of (global) variables for the system. As the system is closed, U is constant and τ is linearly related to t. Yet, the physical interpretation is different in the extreme if U is not zero. Furthermore, it is easy to see that, even if U is zero in one frame, it will not be zero in any other frame which is in relative motion (see the next section). It is clear that τ is uniquely determined by the particles in the system and is available to all observers. Thus it offers a unique definition of simultaneity for all events associated with the global system. On the other hand, if an individual particle is observable, we particle. Furthermore, if a subsystem of particles is observable, the local proper clock of the subsystem offers yet another unique definition of simultaneity (for all events associated with it). Thus, the definition of simultaneity is now unique but depends on the particular convention used.

It should be noted that there is a basic relationship between the global system clock and the clocks of the individual particles. In order to derive this relationship, we return to our definition of the global Hamiltonian K and letW be any observable.
Then,

$$\frac{dW}{d\tau} = \{K, W\} = \frac{H}{Mc^2}\{H, W\} = \frac{H}{Mc^2}\sum_{i=1}^{n}\{H_i, W\}$$

$$(20) \qquad = \frac{H}{Mc^2}\sum_{i=1}^{n}\frac{m_i c^2}{H_i}\left[\frac{H_i}{m_i c^2}\{H_i, W\}\right] = \sum_{i=1}^{n}\frac{Hm_i}{MH_i}\{K_i, W\}.$$

Using the (easily derived) fact that $d\tau_i/d\tau = Hm_i/MH_i = b_i/b,$ we get

$$(21) \qquad \frac{dW}{d\tau} = \sum_{i=1}^{n}\frac{d\tau_i}{d\tau}\{K_i, W\}.$$

Equation (21) allows us to relate the global systems dynamics to the local systems dynamics and provides the basis for a direct approach to the quantum relativistic many-body problem using one (universal) wave function.

We now show directly that the transformation, at the global level, is a canonical change of variables (time).

Theorem 2. There exists a function $S = S(\{\mathbf{x}_i\},\ \{\mathbf{p}_i\},\ \tau)$ such that

$$\mathbf{P}\cdot d\mathbf{X} - H\,dt \equiv \mathbf{P}\cdot d\mathbf{X} - K\,d\tau + dS,$$

$$\sum_{i=1}^{n}\mathbf{p}_i\cdot d\mathbf{x}_i - \sum_{i=1}^{n}H_i dt \equiv \sum_{i=1}^{n}\mathbf{p}_i\cdot d\mathbf{x}_i - K\,d\tau + dS.$$

Proof. set $S = [Mc^2 - K]\tau$. An easy calculation, using the fact that both Mc^2 and K are conserved quantities, shows that $dS = [Mc^2 - K]d\tau$. An additional easy calculation gives the result.

It should be observed that, in a manner similar to that of Horwitz and Piron [22],we can formulate a dynamical principle, which generalizes the Hamilton’s principle, by using the integral invariant of Poincar´e-Cartan (see Arnol’d [23]):

$$I = \oint_{\mathbf{C}}\sum\nolimits_{i=1}^{n}\mathbf{p}_i\cdot d\mathbf{x}_i - K\,d\tau,$$

where C is a closed curve on extended phase space $\Gamma = \Gamma(\{\mathbf{x}_i\},\ \{\mathbf{p}_i\},\ \tau),$ and the above integral is invariant for arbitrary deformations of C along trajectories corresponding to solutions of the equations of motion. A fundamental conclusion of this section is that, for any system of particles, we can always choose a unique observer-independent measure of time that is intrinsically related to the local clocks of the individual particles. (This is not true for any of the attempted formulations of either a classical or quantum relativistic many-body theory.) One important consequence of this result can be stated as a theorem.

Theorem 3. Suppose that the observable universe is representable in the sense that the observed ratio of mass to total energy is constant and independent of our observed portion of the universe. Then the universe has a unique clock that is available to all observers. The above assumption is equivalent to the homogeneity and isotropy of the energy and mass density of the universe.

In the study of physical systems one is sometimes not interested in the behavior of the global system, but only in some subsystem. The cluster decomposition property is a requirement of any theory purporting to be a possible representation of the real world. Basically this is the property that, if any two or more subsystems become widely separated, then they may be treated as independent systems (clusters).

Theorem 4. Suppose that our system of particles can be decomposed into two or more clusters. Then there exists a unique (local) clock and corresponding canonical Hamiltonian for each cluster.

Proof. We assume that the subsystems are sufficiently separated that all observers can agree that they are distinct. In this case, each observer can identify effective masses $M_1,\ M_2$ and Hamitonians $H_1,\ H_2.$ It follows that $d\tau_1 = [(M_1c^2)/H_1]dt$ and $d\tau_2 = [(M_2c^2)/H_2]dt,$ so that each observer can construct a local-time theory for each cluster.

Actually, the theorem is true without the assumption that the systems are weakly interacting. This makes the theorem less difficult to apply than the various phenomenological approaches, which require both model justification and consistency analysis prior to use. This theorem also allows us to prove a weaker version of Theorem 4, in the sense that we replace the assumption of homogeneity and isotropy of the energy and mass density for a possible infinite universe by finite energy and mass density for a possibly inhomogeneous universe.

Theorem 5. Suppose the universe has finite mass and energy density and that each observer can choose a local inertial frame from which his/her region of the universe is at rest relative to the observed system. Then there exists a unique proper clock for the universe.

Proof. Applying the cluster decomposition theorem, our observer can identify masses

M_1 for his/her region of the universe and M_2 for the complement region, along with Hamiltonians H_1 and H_2 It follows that with $H = H_1 + H_2,\ M = M_1 + M_2$ and $d\tau = [(Mc^2)/H]dt,$ define the total mass, Hamiltonian and proper clock for the universe. We can now construct our canonical proper-time Hamiltonian K. Since M and H are fixed, and invariant for all observers, we see that both K and τ are unique and invariant for all observers.

Remark 6*. It should be remarked that* M_i *and* $H_i,\ i = 1, 2,$ *will vary with observers reflecting the non-uniqueness of inertial frames.*

It has been known since the pioneering work of Penzias and Wilson [24], that a unique preferred frame of rest exists throughout the universe and is available to all observers. This is the $2.7\ ^{\circ}\mathrm{K}$ microwave background radiation (MBR) which was discovered in 1965 using basic microwave equipment (by today's standards). This radiation is now known to be highly isotropic with anisotropy limits set at 0.001%. Futhermore, direct measurements have been made of the velocity of both our Solar System and Galaxy through this radiation (370 and 600 km/sec respectively, see Peebles [25]). One can only speculate as to what impact this information would have had on the thinking of Einstein, Lorentz, Minkowski, Poincar´e, Ritz and the many other investigators of the early 1900's who were concerned with the foundations of electrodynamics and mechanics. The importance of this discovery for the foundations of electrodynamics in our view is that this frame is caused by radiation from accelerated charged particles. This frame has not found a natural place in current theoretical models. However, Glashow and co-workers have used it as a part of a program to explore possible departures from strict Lorentz invariance in the context of elementary-particle kinematics (see Cohen and Glashow [26], and cited references). We should note that, since all inerital reference frames are equivalent, the one chosen by any observer is a convention. If we seek simplicity in representations of physical reality, we can attach our frame to the MBR and use the proper-time of the universe for our global clock. In this case, we could satisfy the two postulates of the special theory, while the field and particle equations of any system would be invariant under the action of the Lorentz group (for all observers).

3.1. Time Reversal Noninvariance. We focus on the single particle case. (The same discussion applies to the many-particle case.) Since $d\tau = (mc^2/H)dt$ and, as $K = [H^2/2mc^2 + mc^2/2]$ and m are always positive, we see that if $t \to -t$ (time reversal) or $H \to -H$, then $K \to K$ is invariant, while $\tau \to -\tau$. *Thus, our theory is noninvariant under time reversal at the classical level and, since* τ *is monotonically increasing, we acquire an arrow for (proper) time.* It is thus natural to interpret anti-matter as matter with its proper-time reversed. A more complete (and elegant) discussion requires the introduction of Santilli's isodual numbers [27], in which the unit 1 is replaced by −1 and $ab \to a * b = -ab$so that $(-1) * (-1) = -1$ (see [21]). Thus, by introducing a symmetric theory of numbers, we can construct a completely symmetric theory of matter which avoids all of the natural objections to hole theory, while maintaining consistency with our physical sense of a monotonically increasing time variable. Both Feynman [28] and Stueckelberg [29] introduced the idea of representing anti-matter as matter with its time reversed. Our final conclusion is the same as theirs. However, the two approaches are distinct. In our approach, we replace t by τ and acquire K as its canonical Hamiltonian, so all physical interpretations only require information about τ. The quantum theory now follows by replacing the Poisson bracket in equation (21) by the Heisenberg bracket, which leads to Schr¨odinger-like equations:

$$i\hbar\frac{\partial\psi}{\partial\tau} = K\psi, \text{ and } i\hbar\frac{\partial\psi}{\partial\tau_i} = K_i\psi,$$

for the same (universal) wave function ψ. Since *K*, K_i are both positive definite, the problems which caused confusion during the early attempts to merge quantum mechanics and the special theory of relativity do not arise. The question of particle number is easily included (even in the classical case) by observing that, for any closed system of interacting particles, we can replace the definite particle number *n* by a variable (random) particle number *N(t),* the number of particles up to time t (as seen by the observer), with the constraint that the total global energy, momentum, angular momentum and spin remain constant and, as in QED, for large negative $t,\ N(t) \to n_i,$ (the initial particle number), and for large positive $t,\ N(t) \to n_f$ (the final particle number). Reflecting on the possibility of a big bang at the start of the universe, it is now natural to assume that such an event began with conservation of energy, momentum etc, with matter moving forward in time, while antimatter moved backward in time to another (inaccessible) portion of the universe. This would give us a symmetrical theory of matter and explain the lack of large amounts of antimatter in our portion of the universe.

4. LIGHT: ITS NATURE, ITS MASS AND ITS SPEED

4.1. Photon Nature. Given that in our local-time formulation of the Maxwell theory, the value of the "natural" speed of light depends on the motion of the source, it is not presumptuous to take another look at the age-old problem of its nature and seek to understand the impediments to treating a photon on the same footing as any other elementary particle. The single most important problem with any attempt to treat photons as elementary particles is its well-known wave property. Interference and diffraction experiments and, indeed, a number of major fields of electrical engineering attest to the amazing precision, effectiveness and efficiency of the wave picture. The field theory view of both photons and particles is that they are localized wave packets of the quantized electromagnetic field (or the particular particle field). Hence, one expects that matter will behave like particles in some experiments and like waves in others and, as such, may be viewed as some type of atomic compromise between two distinct classical views. However, these ideas go back to the fundamental work of Born, deBroglie and Heisenberg while today, we have much more experimental information about photons since those pioneering efforts. It has now been known for over twenty five years that we can control the intensity of a beam of photons in interference (or diffraction) experiments to the point that individual photons may be counted on a photographic plate (see Paul [30]). Furthermore, the distribution of photons falling on the plate appears random and, only after a long time period (depending on the intensity level), do we begin to see wave patterns. These experiments, along with the photoelectric and Compton effects, conclusively tell us the following:

1) It is not at all unreasonable to treat photons as elementary particles.
2) The wave length and frequency we attach to individual photons are actually characteristic of groups of photons, which represent correlated properties derived from their common source.
3) Concepts in the Maxwell theory such as electric and magnetic fields are macro-properties that have some (but limited) reality on the atomic-scale and (possibly) none at the sub-atomic-scale. For an excellent discussion of the above and an important correction to Newton's corpuscular theory of light, see Buenker [31] and related papers (which may be found in the archives (e.g., lanl.arXiv.org)).

4.2. Photon Mass. In the past, work on the question of photon mass has focused on the addition of a mass term to the Lagrangian density for Maxwell's equations and generally leads to the Proca equation (see Bargmann and Wigner [32]). Early work in this direction can be traced back from the paper of Schr¨odinger and Bass [33]. As in our approach, the speed of light is no longer constant in all reference frames. In this case, the fields are distorted by the mass term and experiments of Goldhaber and Nieto [34] use geomagnetic data to set an upper bound of $3 \times 10^{-24}\, GeV$ for the mass term (see Jackiw [35]). This approach causes gauge problems, and has not found favor at the classical level. From our derivation of the wave equations, it is clear that the local-time theory is fully gauge invariant and the (photon) mass is dynamical, appearing only during acceleration of the source. (As noted by Feynman [38], a small photon mass will eliminate the infrared catastrophein QED.) It should be recalled that Maxwell's equations are (spin 1) relativistic wave equations (see Akhiezer and Berestetskii [36]). On the other hand, the experiments of Pound and Snider [37] show directly that photons have an apparent weight (as one would expect of any material object). These experiments do not depend on either the special or general theory of relativity and are not directly dependent on frequency or wavelength measurements.

4.3. Light Speed. In this section, we give additional consideration to the physical implications of our interpretation of b and b' as the speed of light relative to the source for the different observers (collaborative speed of light). In order to gain some perspective, suppose an emitting system is at rest in the unprimed frame so that $b = c$. In this case, the collaborative speed of light observed in the primed frame is $b' = \gamma(\mathbf{v})c$ and the velocity of the source is seen as $\mathbf{u}' = -\gamma(\mathbf{v})\mathbf{v}$. Thus, if the two observers are separating at high speeds, both b' and $\mathbf{u}'$ may be very large. There are some experiments where use of the observer's clock provides a clear answer. A classic example is the Michelson-Morley experiment. This experiment gave the first bell of doom for the ether theory, and is easily explained by the special theory (using the clock of the observer). It also has a simple explanation when the clock of the source is used since; in this case, the source is at rest in the frame of the observer so that $\mathbf{u} = 0 \Rightarrow b = c$. It is clear that, at the speeds obtained in the world of our ordinary experience, no significant difference between the two approaches is expected. However, at high energies, we expect differences to show up in a dramatic way. Indeed they have, but our definition of velocity depends on the clock attached to the observer, $\mathbf{w} = d\mathbf{x}/dt$, , and all contrary results are interpreted as due to time dilation. Indeed, without this switch in clocks, there is no way to explain the results. An equally valid interpretation is that the velocity of the system is not w, but
$\mathbf{u} = d\mathbf{x}/d\tau$ and, in this case, no contrary results occur. The use of w is clearly a convenient choice for most of ordinary physics (where both choices are the same). However, in high-energy experiments, the local clock of the system is necessary (and used) to determine both when and where to set up particle detectors to record scattering events. The data is then analyzed using time dilation to make the results correspond to velocities below *c*. In order to obtain a different view of experiments on the lifetime of fast mesons and the velocity of rays and light from moving sources, first consider the definition of momentum. When the clock of the observer is used to measure time, momentum increase is attributed to relativistic mass increase so that

$$\mathbf{p} = m\mathbf{w}, \quad m = m_0[1 - w^2/c^2]^{-1/2}.$$

On the other hand, if we use the clock of the source, we have that

$$\mathbf{p} = m_0\mathbf{u}, \quad \mathbf{u} = \mathbf{w}[1 - w^2/c^2]^{-1/2}$$

So that there is no mass increase, the (proper) velocity increases. Thus, in particle experiments, the particle has a fixed mass and invariant decay constant, independent of its velocity, but can have speeds > c. An analysis of experiments on the lifetime of fast mesons, the velocity of rays and light from moving sources reveal that, at some point, either the speed of light is assumed to be independent of the motion of the source, or time dilation is used. Both assumptions imply that the clock of the observer is used. Thus, these experiments validate the conventional theory but do not prove that the speed of light is *c*.

Conclusion

In this paper, we have provided an outline of the research program at Howard University on our approach to a physically motivated representation of classical electrodynamics. The following points are of special notice:

1) There is a formulation of the special theory of relativity in which the invariant speed of light c is replaced by the invariant local clock of the observed system. In this formulation, the new (collaborative) speed of light is not invariant but depends on the motion of the observed system.
2) There is a formulation of the special theory of relativity in which the local metric is defined in three-dimensional Euclidean space and becomes deformed in the presence of a potential field.
3) There is a formulation of the special theory of relativity in which simultaneity is unique but depends on a convention.
4) There is a formulation of the special theory of relativity which fixes a particulartime direction (e.g., is non-invariant under time reversal).
5) There is a formulation of Maxwell's equations that is mathematically but not physically equivalent to the conventional one. (Thus, mathematical equivalence is not necessarily physical equivalence.)
6) There is a formulation of classical electrodynamics that does not depend on the structure of charged particles and does not require self-energy, advanced potentials, mass renormalization, or the problematic Lorentz-Dirac equation in order to account for radiation reaction.
7) All experiments and observations based on the assumed constant speed of light *c* need re-evaluation.

In light of the above, it is no longer clear how far cosmic rays can travel, how far we are from the distant galaxies, nor how old the universe is. Our current research efforts are directed toward studying the canonical propertime version of the long neglected spin- $\frac{1}{2}$ representation of the square-root equation. Recall that, failure to understand this equation led to the Dirac equation as an alternative.

In [39], we have used the theory of fractional powers of linear operators (developed by researchers in probability theory) to construct a general (analytic) representation theory for the square-root energy operator of relativistic quantum theory which is valid for all values of the spin. Our general representation is uniquely determined by the Green's function for the corresponding Schr¨odinger equation. We find that, in general, the operator has a representation as a (spacial) nonlocal composite of, at least, three singularities (divergent integrals). In the standard interpretation, the particle component has two negative parts and one (hard core) positive part, while the antiparticle component has two positive parts and one (hard core) negative part. This effect is confined within a

Compton wavelength such that, at the point of singularity, they cancel each other providing a finite result. Furthermore, the operator looks like the identity outside a few Compton wavelengths (cutoff). To our knowledge, this is the first example of a physically relevant operator with these properties. (It was shown in [40], that the square-root operator is related to the Dirac operator by a unitary transformation, but one is spatially nonlocal, while the other is time nonlocal. Thus, also in this case, mathematical equivalence is not physical equivalence.)

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(Tepper L. Gill) Department of Electrical Engineering Howard University, Washington DC 20059, USA, E-mail : tgill@howard.edu
(Woodford W. Zachary) Department of Electrical Engineering, Howard University, Washington DC 20059, USA, E-mail : wwzachary@earthlink.net.

DEVELOPING COMPUTATIONAL PHYSICS IN SOUTH AFRICA

Nithaya Chetty

University of KwaZulu-Natal

Pietermaritzburg

UNIVERSITY OF KWAZULU-NATAL

Two points

- Intra-African collaborations in all academic disciplines must grow substantially
- Computational Physics Research and Teaching is a very practical means of enhancing intra-African collaborations

Intra-African collaborations

- Africans need to collaborate on African problems
  - Historical and Geographical connection
  - Health, poverty alleviation, agriculture, food security, mathematics literacy, climate change, transport, etc
- Africa is a very, very big continent
  - Multi-lingual, multi-cultural, poor infra-structure, poor communications, low understanding and commitment to basic Science and Technology

Opportunities

- Lots of goodwill internationally for scientific development on the African continent
  - e.g. IUPAP, ICSU, Bi-national agreements
- Recognition of the importance of science as an instrument for development
- Correlation of basic scientific literacy and democracy

Challenge for the Scientific Community

- Find more direct ways to impact on real life challenges in Africa
- Become champions for the public understanding of science
- Interface with government, commerce and industry to argue for the importance of scientific development
- There is a huge responsibility on our generation to strengthen the base for the scientific development for the future

Burgeoning Pan-African initiatives (within Physics)

- African Materials Research Society
- African Laser Centre
- African Institute for Mathematical Sciences
- National Institute of Theoretical Physics
- Southern African Large Telescope

Developing Computational Physics

Outline

- What is computational Physics? What is computational Sciences? Is this an academic discipline? How is this distinguished from Computer Science?
- What is the state of Computational Sciences in South Africa – at our universities (both undergraduate and postgraduate levels), our national laboratories, in government, commerce and industry?
- How can we improve the culture of Computational Sciences in the country? Why has this become an urgent matter? Report on Computational Physics Working Group.
- How can we develop a more fundamental approach to Computational Sciences? How have we proceeded at UKZN?

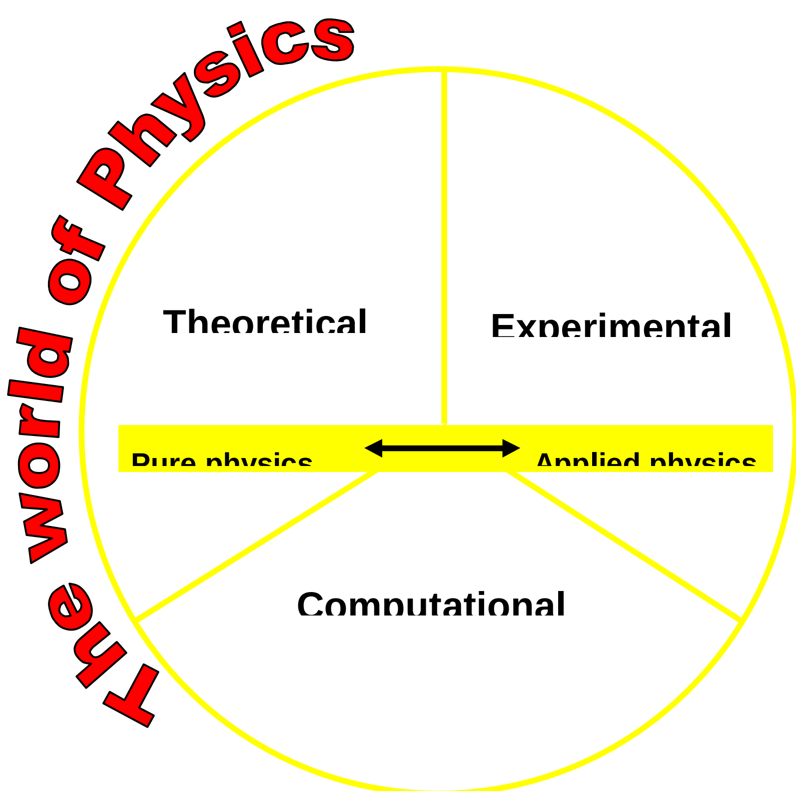
Appendices

- Developing Computational Mathematics in Africa, By N. Chetty and A.C. Bawa, What Mathematics from Africa? Publisher Polimetrica, Editor Giandomenico Sica, 2005
- Computational Physics in South Africa, By N. Chetty, F. Petruccione and R. J. Lindebaum, South African Journal of Science, 2005
- “Developing a strategy for Computational Physics in South Africa” report of the CPWG to be published December 2008
- Flyer on Computational Physics @ UKZN

What is Computational Physics?

Computational Physics is a (relatively) new mode of studying physics. It is sandwiched between theoretical and experimental physics, and focuses on the practical use of computers in solving physical problems. Computational Physicists pay serious attention to the detailed mathematical, physical, algorithmic, programming and graphical aspects of the problem at hand, and are often interested in the optimal use of the computing hardware and software resources

Distinctly different from Computer Science!



Condensed Matter Physics

Solid State Physics

Particle Physics

Nuclear Physics

Radiation Physics

Astronomy

Astrophysics

Fluid Dynamics

Complex Systems

Cosmology

Space Physics

Plasma Physics

Optics

Spectroscopy

Biophysics

Computational Approach

- Apply physical principles
  - Quantum mechanics, classical mechanics, statistical physics, electrodynamics, fluid mechanics
- Model the system, i.e. describe it mathematically
  - E.g. differential equations, integral equations, algebraic equations, complex variables, matrices, random numbers
- For realistic problems, the solutions are usually intractable! Need to compute!

Facets of Computational Physics

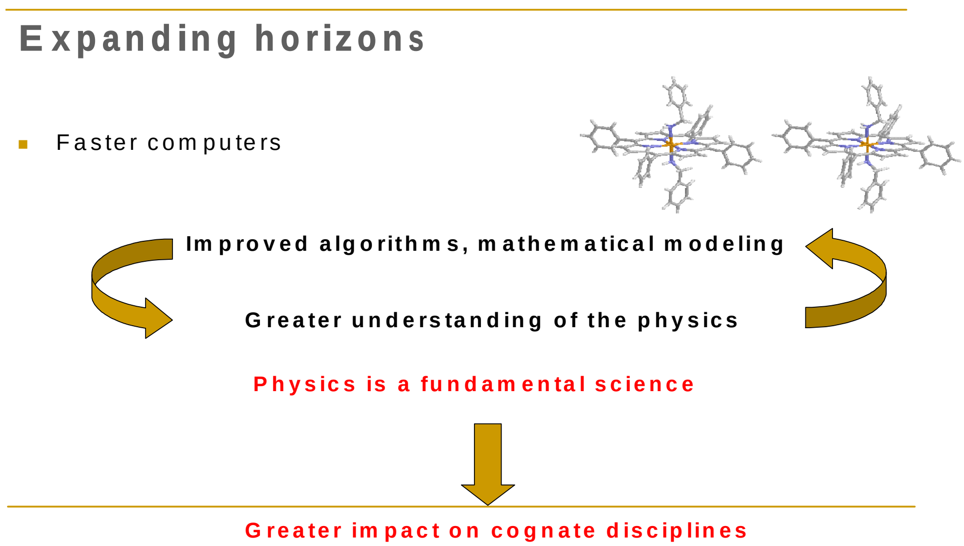
- Algorithmic development
- Numerical methods
- Programming
- Graphical work

Applied to physics of real problems

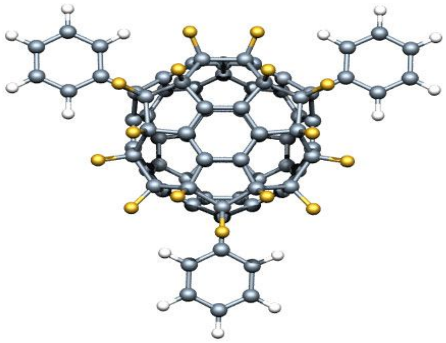
Virtues of Computational Physics

- Computational Physics expands the boundaries of Physics
- Skills transferable to other cognate disciplines
    - Computational Chemistry
    - Computational Biology
    - Genetics
    - Engineering
- In so doing, students learn useful transferable skills that enhance their employment opportunities

Expanding horizons

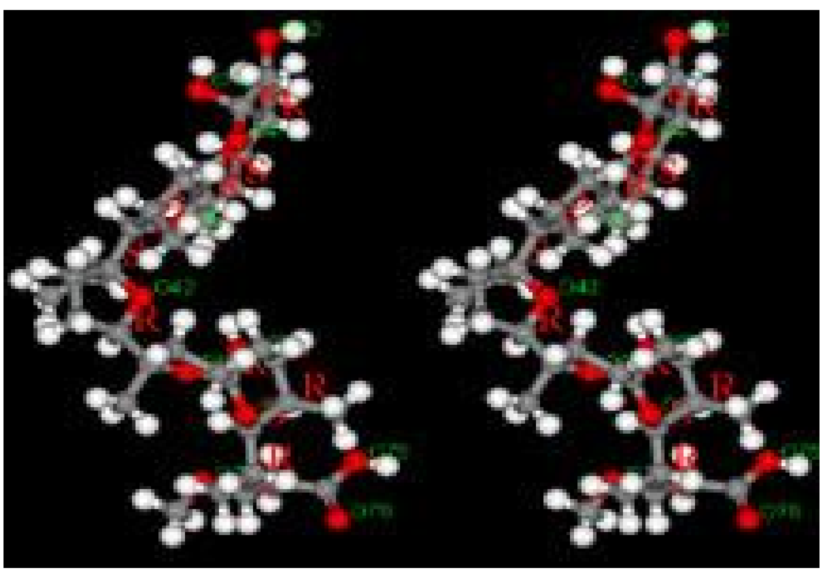


Computational Chemistry



- Quantum mechanical studies
    - Density functional theory, improvements to exchange-correlation functionals, eg. GGA
    - Molecular systems, excited states, dynamical processes
- Within desired accuracy of milli-Calorie/mol accuracy

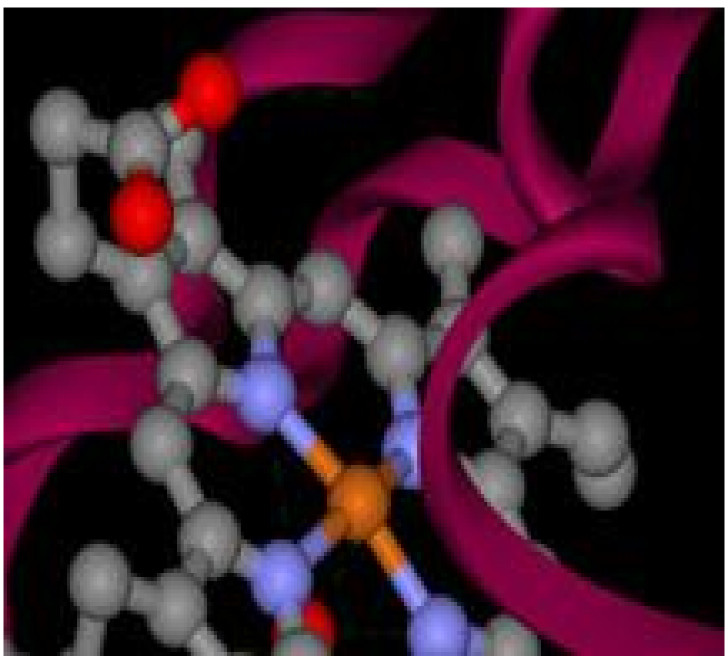
Walter Kohn received the 1998 Nobel prize in Chemistry



Nanoscience

- Exciting new frontier of science
- Important new knowledge and applications
- Merging of classical and quantum worlds
- Merging of physical and biological worlds

Biological Physics

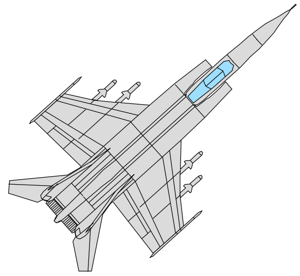


Computational modeling of biological macromolecules

- Protein folding
- Enzyme action
- Catalytic processes

- Classical Modeling
- Quantum Mechanical Modeling
- Stochastic modeling

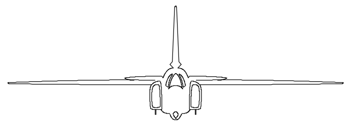
Engineering applications



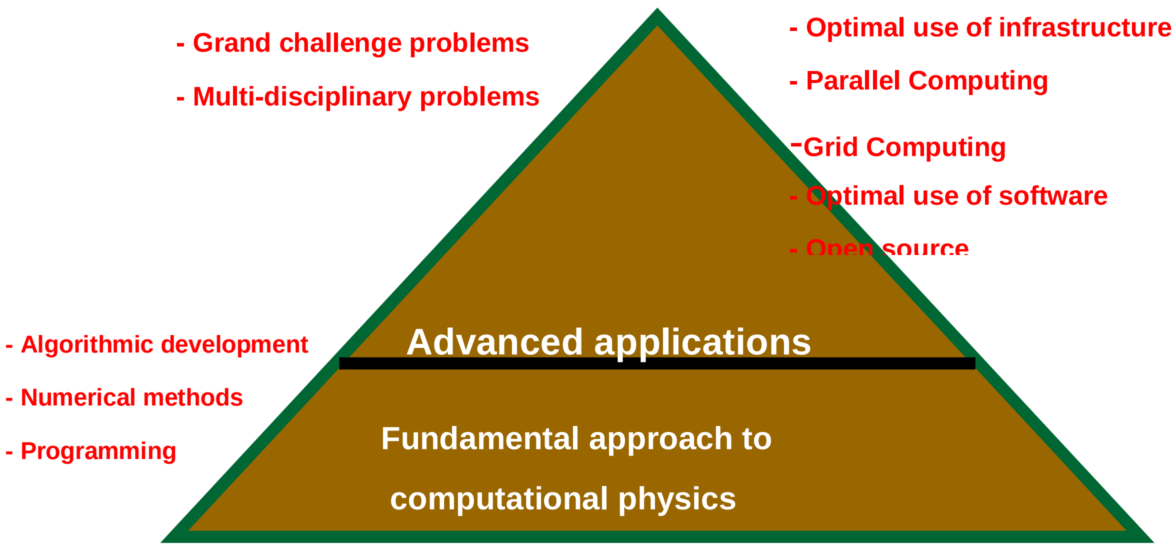
- Material constitutive relations
- Mechanical models
- Aero-space applications
- Fluid flow problems

State of Computational Sciences in South Africa

- `We don't have a strong culture of computer code development in SA
    - Too much of reliance on ready-to-go packages
    - Expensive licensing costs
    - Questionable relevance for South African solutions
    - Too much of reliance on Windows systems



- Computational Physics community is too fragmented



South African Institute of Physics - Computational Physics Working Group

and

Centre for High Performance Computing – Computational Physics Special Interest

Group

SAIP-CPWG/CHPC-CPSIG
Established in 2006

A Strategy to Develop Computational Physics in South Africa

- Members of CPWG
  - Brandon van der Ventel (Stellenbosch)
  - Rubin Landau (Oregon State)
  - Daniel Joubert (WITS)
  - Roger Fearick (UCT)
  - Habatwa Mweene (Zambia)
  - Igle Gledhill (CSIR)
  - Jan Groenewald (AIMS)
  - Jean Cleymans (UCT)
  - Jeff Chen (CHPC)
  - Robert Lindebaum (UKZN)
  - Marius Potgieter (North West)
  - Morits Braun (UNISA)
  - Francesco Petruccione (UKZN)
  - Lutz Ackerman (Limpopo)
  - Thuto Mosuang (Limpopo)
  - Nithaya Chetty (UKZN) - Convenor

Outcomes

- Review: Better understanding of state of Computational Sciences in South Africa
- Recommendations: Coherent view of where we would like to go to and how to get there
  - Government
  - Funding agencies
  - Higher education institutions
  - National laboratories
  - Industry
  - Other professional societies
  - International bodies
  - Physics community

Impact

- More international quality conferences, workshops in Computational Physics
- More international collaborations
- More quality research and teaching in Computational Physics
- More investment in local computational resources, people, students
- More optimal use of computational resources
- More migration of computational graduates to other cognate disciplines
- More impact on industry

Report – Table of Contents

Acknowledgements
Executive summary

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Chapter 2 – Why is Computational Sciences important for SA?

Chapter 3 – Why is Computational Physics important for SA?

Chapter 4 – Enhancing Research in Computational Physics

Chapter 5 – Teaching of Computational Physics

Chapter 6 – Outreach

Chapter 7 – Industrial participation

Chapter 8 – African participation

Chapter 9 – Recommendations

References

Appendices

Teaching of Computational Physics

- Curriculum and content development
- Skills at appropriate levels
- Assessment
- Texts and references

- Issues around plagiarism

School of Physics UKZN

- Westville Campus – Physics, Applied Physics
- Howard College Campus – Engineering service teaching
- Pietermaritzburg campus – Physics, Computational Physics

PMB is the only centre in the country that offers BSc(CompPhys)

BSc (CompPhys) @ UKZN 1st year

- 1st Semester
  - PHYS110 (16C) Physics
  - MATH130 (16C) Maths
  - COMP100 (16C) Computer Science
  - 16C chosen freely
- 2nd Semester
  - PHYS120 (16C) Physics
  - MATH140 (16C) Maths
  - COMP102 (16C) Computer Science
  - 16C chosen freely

BSc (CompPhys) @ UKZN 2nd year

- 1st Semester
  - PHYS211 (16C) Physics
  - PHYS231 (16C) Computational Physics Techniques
  - 8C of Maths
  - 24C chosen freely
- 2nd Semester
  - PHYS212 (16C) Physics
  - CPHY212 (16C) Computational Classical Mechanics
  - 8C of Maths
  - 24C chosen freely

BSc (CompPhys) @ UKZN 3rd year

- 1st Semester
  - PHYS306 (16C) Quantum Mechanics, Statistical Physics
  - PHYS351 (16C) Spectroscopy, Classical Mechanics, Expt
  - CPH311 (8C) Computational Quantum Mechanics
  - CPH321 (8C) Computational Statistical Physics
  - 16C chosen from Maths, Computer Science or Statistics
- 2nd Semester

- PHYS305 (16C) Electromagnetism and Solid State Physics
- PHYS352 (16C) Modern Physics, Expt
- CPHY312 (8C) Advanced Computational Statistical Physics
- CPHY322 (8C) Computational Solid State Physics
- 16C chosen from Maths, Computer Science or Statistics

BSc Honours (Phys) @ UKZN

- Structure of Honours programme is flexible
    - Additional Computational Physics modules
    - Computational Physics projects
    - Visiting scholars, e.g. NITheP
    - Industrial internships
    - Remote supervision

Graduate studies

- Computational Masters and Doctoral studies
    - Molecular physics, nuclear physics, particle physics, condensed matter physics, solid state physics, cosmology, astrophysics, biological physics, etc, etc
- Experimental physics too!!

Other models for CPHY

- Integrate CPHY into mainstream physics laboratories
- Develop joint programme with physics and mathematics
    - BSc Honours
    - MSc course work + thesis
- Involve other cognate disciplines
    - Computer Science, Statistics, Chemistry, Geography, Earth Sciences, GIS
- Named programme
    - Computational Physics, Computational Mathematics, Computational Mathematical Physics, Computational Sciences, Scientific Computing, Computational Physical Sciences

Computational hardware facility

- A LAN of about networked 30 computers is needed
    - We secured sponsorships from de Beers, Shuttleworth, UKZN
    - Need an upgrade strategy, keep with changing technology
- Control
    - Internet usage

    - Printing
    - Use of resources for other work, especially private work
    - Time management
- Air-conditioning, security, after hours access

Systems administration

- Linux is essential
- Systems administration
    - User accounts
    - Disk management
    - Networking
    - Printing
    - Scanning, etc
    - Upgrading
    - General trouble shooting

Computational Software Management

- Programming languages
    - Fortran 90
    - Mathematica
    - C++, Java, VPython
    - Graphical programmes – 2D and contour plots
    - Graphical visualization, animation

Assessment – Tests

- Difficult to set examinations
- Continuous assessment is mandatory
- Test short computer code development
    - Write short subroutines
    - Correct a defective program

- Tests involving pen on paper
    - Pay less attention to syntax, more to algorithm development, logic, computational efficiency

Assessment – Short submissions

- Scrutinize the code
    - Does it work?

  - Test the code. Helpful to do this with the student.
  - Request real-time changes to the code
  - Check structure, comments, algorithm, logic, computational efficiency
  - Oral examinations at the computer workstations
- Results
- Graphical presentation
- Explanation of results

Intra-African Computational Physics Collaborations

- Easy to do over the network
- Cheap and effective research and education
- Access to hardware facilities, e.g. Centre for High Performance Computing in Cape Town
  - IBM Blue Gene
  - Grid Computing
- Applications to Grand Challenge problems
- Students learn practical transferable skills