MATHEMATICAL MODELING: AN APPLICATION TO CORROSION IN A PETROLEUM INDUSTRY*

Oyelami, Benjamin Oyediran[1]
&
Asere, Abraham A[2]**.**
1. Mathematical Sciences Programme
2. Petroleum Engineering Programme
Abubakar Tafawa Balewa University
Bauchi – NIGERIA

Abstract

Mathematical modeling is richly endowed with many analytic computational techniques for analyzing real life situations. Recent reports have confirmed that several billon dollars were lost to corrosion, in addition to environmental pollution and economic wastage in cleaning up the environmental mess caused by corrosion. This paper considers application of mathematical modeling to corrosion problems. It uses the mathematical modeling techniques to forecast the life expectancy of industrial equipment in the refinery, petroleum reservoirs and gas pipelines' distribution. The models considered in this direction are the heat-mass transfer equation, Zhim-Hoffman's equation, equations arising from electrolysis and finally gas pipeline distribution.

Keywords and Phrases:

Mathematical models, corrosion, phase transition, heat equation, galvanic corrosion.

1. Introduction

Mathematical modeling is as old as mathematics and has extended its tentacles to unforeseeable directions. Mathematical modeling can best be described as a sandwich between mathematical theory and applied mathematics. It is an encyclopedia of theories and techniques as applicable to real life situations.

Corrosion in the modern society is one of the outstanding challenging problems in the industry. Most industrial design can never be made without taking into consideration the effect of corrosion on the life span of the equipment. Recent industrial catastrophes have it that many industries have lost several billion of dollars as a result of corrosion. Reports around the world have confirmed that some oil companies had their pipeline ruptured due to corrosion, oil spillages are experienced which no doubt created environmental pollution, in addition, resources are lost in cleaning up this environmental mess and finally large scale ecological damage resulted from corrosion effects.

* AMS subject classification:73V & 80A

The possibility of occurrence of corrosion in an industrial plant has been posing a lot of concern to Petroleum, Chemical, Mechanical Engineers and Chemists. It is now known that corrosion can have some effects on the chemistry of a chosen process, and product of corrosion can affect reaction and purity of the reaction products.

Furthermore, it is also true that a lot of profit can run out of a hole in any industrial plant if care is not taken, but early advice from corrosion experts can prevent that from happening. The study of corrosion is multi-disciplinary in nature. Its calculation involves knowledge of viscosity, specific heat capacity, thermal conductivity and density of the fluid concerned, in some cases, a thorough study of the property of the material from which the plant is fabricated is highly required. The present paper examines the effect of corrosion on equipment in a typical oil industry. The study elucidates the application of mathematical modeling to corrosion problems in the oil industry.

The justification or the motivation for this study is conceived from the following facts:

1. In the oil refinery, fractional distillations of crude oil into fractions are performed in perforated trays. There could be contamination of the products by corrosion or impurities arising from the surface as the plants age with time.

2. Corrosion may attack pipeline of crude and refined oil.

3. Most petroleum products are stored in varieties of containers and oil reservoirs whose materials may be corrosive because of variation in temperature, heat exchange and pressure of the petroleum products as affected by environmental factors.

4. Which materials are less corrosive that could be universally adopted for storing and transporting petroleum products?

5. Corrosive nature of some crude and fractions that have high content of sulphur compounds called mazcaptan or thiols.

Mathematical modeling offers several powerful intuition appealing tools for studying and analyzing the chemical kinetics and the thermo chemistry of compounds (e.g., in petroleum crude and products).

Mathematical modeling also offers quantitative and qualitative techniques for investigating the material science of the industrial plant for producing, storing and transporting the petroleum products.

Before we embark on the formulation of the mathematical models, it is pertinent to make a thorough exposition on mathematical modeling and corrosion itself.

2. What are Mathematical Modelling and its Usefulness?

Mathematical modeling is the act of relating abstract ideas of mathematics to real life problems. The process involves expressing a real life situation into mathematical terms, manipulating the mathematics and translating the mathematical results back into the real life.

It is an undisputable fact that every human activity involves one mathematical problem or the other; the need to use mathematical modelling is increasing released in modern times. It gives us insight into many real life processes and the interplay between or among variable(s) quantifying such models. This process saves cost and labour that would unnecessarily have been expended.

Different researchers have expressed various steps taken to model a problem. The most outstanding one is summarized by Ale (1981; 1986), described the process involved in a modeling a process as follows:

The identification of the real life problem, which involves modifying and simplifying the original problem into a reasonable precise and succinct manner.

To have a full grasp of the idea of modeling, the following flow chart, and fig. 1 states the steps to be taken when modeling a problem. In

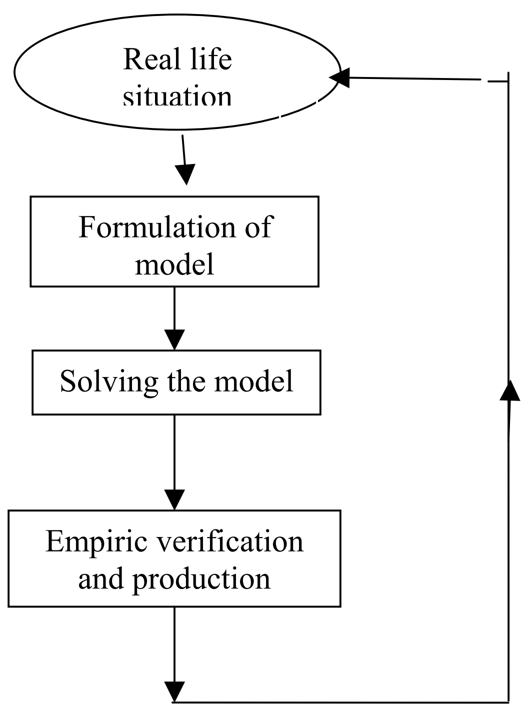


Fig.1: Idealisation of real life problem into a model

In real life, there is the problem whose solution is sought. This problem need to be identified, in which case, the significant features are identified and translated into mathematical entities, leading to the mathematical model. Once a model is constructed, it needs validation, that is:

1. The mathematical structure it represents is self- consistent (i.e., it contains no contradictory statements) and obeys all usual mathematical laws underlying it;
2. It represents the situation it is actually designed for.

Various branches of mathematics have been created in an attempt to solve some problems or the other. One may want to predict the weather, estimate the lost caused by corrosion and so on. If one needs to analyze a problem, it is often a good idea to start by building up a model.

A model is nothing fanciful, it is simply the "bare bones" of the problem - what it looks like after stripping away the unimportant details. The reduced version of the original problem is what model represents. The importance of a model is not far fetched for:

- A model is more reliable than pure intuition.
- Mathematically, a model simplifies the analysis.
- A good model is economical (Morton, 1980). That is, it can be labour - saving devices in more than one way.

A model used for one purpose can also be used for an entirely different purpose.

3. Why Do We Need to Study Corrosion?

Many petrol chemical plants are large-scale equipment, which could be corrosive after some time. Mathematical model to determine the amount of contamination arising from corrosion will have to investigate the deleterious effect of the corrosion on the process of the reaction on the product quality.

The classic example of intergranular corrosion in chemical plant is the weld delay of unstabilized stainless steel. This is caused by the precipitation of chromium carbides at the grain boundaries in a zone adjacent to the weld, where the temperature has been between 500^0C - 800^0C during welding.

Corrosion rate and the form of attack can change if the material is under stress. For some combination of metal, corrosive media and temperature, the phenomenon called stress cracking can occur. This leads to premature brittle failure of the metal that constitutes the petrol chemical plant.

The conditions that cause corrosion can arise in a variety of ways. For the brief discussion on the selection of material, it is convenient to classify corrosion into the following categories:

1. General wastage of material - uniform corrosion;
2. Galvanic corrosion - dissimilar metals in contact;
3. Pitting - localized attack;
4. Intergranular corrosion;
5. Stress corrosion;
6. Corrosion fatigue;
7. Erosion - corrosion;
8. High temperature oxidation;
9. Hydrogen embrittlement.

Metallic corrosion is essentially an electrochemical process. Fair components are necessary to set - up an electrochemical cell:

1. Anode - the corroding electrode;
2. A cathode - the positive, non - corroding electrode;
3. The conducting medium - the electrolyte - the corroding fluid;
4. Completion of the electrical circuit - through the material.

Cathodes areas can arise in many ways:

(i) Dissimilar metals;
(ii) Corrosion products;
(iii) Inclusions in the metal, such as slag;
(iv) Less aerated areas;
(v) Areas of differential concentration;
(iv) Differential strained areas.

Consider the simplest corrosion problem in nature where iron is exposed to the atmospheric oxygen in the presence of moisture leading to formation of rust, iron (III) oxide as well as iron (III) chloride respectively. The chemical reaction can be summarized as follows:

A. $4Fe(s) + 30_2(g) \rightarrow 2\,Fe_2 0_3(s)$ Oxidation State of Fe is 0 → +3

B. $2\,Fe^0(s) \quad + \quad 3\,Cl^0{}_2(g) \;\rightarrow\; 2\,Fe^{+3}\,Cl^{-1}{}_3(s)$

Reducing Agent   Oxidizing Agent

The processes A & B illustrates the importance of redox reaction, i.e. oxidation and reduction processes. In these processes, iron is an oxidizing agent since it gains 3 electrons and chlorine is a reducing agent since it loses 1 election.

From the above example and some other ones found in the literature, we note that corrosion process involves molecular and electronic charge exchanges in such a way that:

* There is energy depletion as a result of redox reactions
* Synthesis of compounds formed as a result of oxidation and some compounds lost as result of reduction.
* The chemistry of corrosion involves complex interactions of compounds in form of chemical reaction activation process.
* Some basic properties of the original material before corrosion takes place such as malleability, luster, conductivity and ductility etc is lost may be after corrosion.
* Knowledge of viscosity, specific heat capacity, thermal conductivity and density of the fluid concerned and even material science of the plant, where the corrosive media is kept, need be paid special attention.
* The prediction of the life expectancy of industrial plant using equation derived need be paid attention.

4. The Mathematical Models

In corrosion testing, the corrosion rate is measured by the reduction in weight of a specimen of known area over a fixed period of time. This is expressed by the formula

$$ipy = \frac{12w}{tAP}$$

Where

w = mass loss in time t/kg
t = time, years
p = density of material, kg/m^3
A = surface area, m^2

In SI units, ipy = 25mm per year.

For material cost, cost-rating equation is given by

$$Cost\ rating = \frac{CX_p}{\sigma_d}$$

Where

CXp = Cost per unit mass, \$/kg
P = density, kg/m^3
σ_d = design stress, N/mm^2.

British standard on corrosion (BS18\$, BS1501) resistant materials made the following classification:

Table 1: Acceptable corrosion rates

	ipy	mm/y
Completely satisfactory	< 0.01	0.25
Use with caution	< 0.03	0.75
Use only for short exposure	<0.06	1.5
Completely unsatisfactory	>0.06	1.5

We start our discussion with corrosion accompanied by mass - heat transfer:

4.1 Mass - Heat Transfer Model

Heat transfer in most industrial plants often accompanies corrosion; hence, we consider the following heat transfer flow in the following diagram:

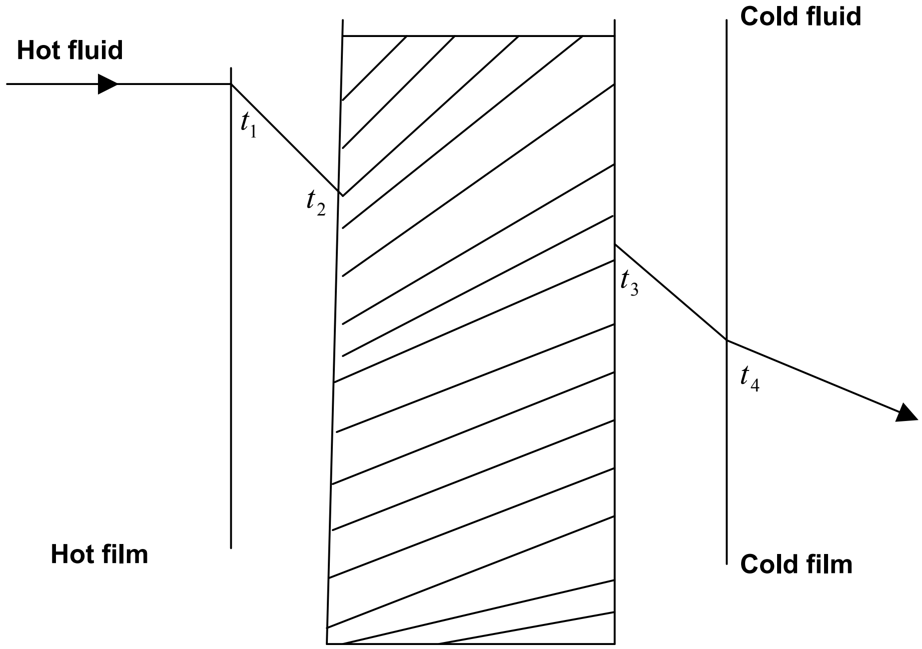


Fig.2: Temperature distribution for heat transfer across a metallic wall

The above figure shows the mechanism of heat transfer across a metallic wall. Three separate resistances are involved. The first is through a film of fluid, liquid or gas, adjacent to the metal walls of the vessel or tube. The other two resistances are (a) The wall of the metal whose resistance is function of the thermal conductivity of the metal as well as its thickness, (b) A firm that forms a thin boundary layer just at the surface of the metal on the other side before the moisture.

The basic heat transfer model is heat lost per unit area = U x (total temperature difference (Berry (1990))

Heat conducted through a material between temperature differences T_1 - T_0 across the surface is

$$Q = \frac{k}{a}(T_1 - T_0) \qquad \text{(Conductivity)}$$

Q is heat lost per unit area; k is thermal conductivity of the material; a is the thickness of the material.

The heat lost by convection using linear model

$$Q = h_1(T_1 - T_0)$$

$$Q = h_2(T_2 - T_1)$$

h_1 and h_2 are constants called convection heat transfer coefficients.

Eliminating T_1 and T_2 between these three equations, we have

$$Q = \left[\frac{1}{h_1} + \frac{a}{k} + \frac{1}{h_2}\right]^{-1}(T_1 - T_0)$$

In general, heat loss across the surface in fig 1, using a simple model is

$$Q = \left[\frac{1}{h_1} + \frac{2k}{a} + \frac{1}{h_1} + \frac{1}{h_2}\right]^{-1}(T_1 - T_0)$$

A more general expression for Q involving the viscosity of the fluid into the model is found in the literature and is

$$Q_a = \left[\frac{1}{h_1} + \frac{2k}{a} + \frac{1}{h_1} + \frac{1}{h_2} + \xi(\mu)\right]^{-1}(T_1 - T_0)$$

$\xi(\mu)$ is a function depending on μ, the viscosity.

Of what practical importance is the above heat transfer equation is to corrosion if expressed in terms of temperature differences? For example, the generation of heat across the surface is closely related to that of a laser - drilling appliances. This is found in Beddling (1994).

A laser impinges on a material to be drilled (usually a metal) causes vaporization of the material and result in a moving boundary as a hole formed.

Beddling emphasized that various approximations and simplifications may be made, including that regarding the hole are one - dimensional.

The governing heat equation for the laser - drilling equation is

$$\frac{\partial^2 U}{\partial \varepsilon^2} + \frac{\partial U}{\partial \varepsilon} + \frac{\partial U}{\partial T} = 0$$

Where

$U(\varepsilon, T)$ is (dimensionless) temperature T and ε dimensionless is boundary position respectively.

The boundary conditions are

$$u(\varepsilon,0) = 0, \qquad \varepsilon > 0$$
$$u(0,T) = 1, \qquad T \geq 0$$
$$u(\infty,T) = 0, \qquad T \geq 0$$

ξ is actually Z - T where Z is the momentary position of the boundary. The solution to the model is found by Bedding as

$$u(Z,T) = \frac{1}{2}\{ erfc\,(\frac{z}{2\sqrt{T}}) + \exp(-Z - T)\, erfc\,(\frac{(2 - 2T)}{2\sqrt{T}})\}$$

In relation to heat dissipation, Q_a

$$Q_a = \left[\frac{1}{h_1} + \frac{2k}{a} + \frac{1}{h_c} + \frac{1}{h_2} + \xi(\mu)\right]^{-1} \left[u(z,T_1) - u(z,T_2)\right]$$

Where

T_1 and T_2 are two times for heat to pass across the metallic surfaces.

The quantity of heat generated in the plant can be monitor-using computer or a mathematical machines given that the temperatures $u(z,T_1)$ and $u(z,T_2)$ are known.

To calculate the life expectancy of the plant as a result of corrosion, we use the idea of perturbation theory. Since the thickness of the surface before corrosion takes place, is Q and after corrosion, a^1 we assume that the thickness is a. Then

$$a = a^1 + h$$

h is the part lost into the chemical reaction as a result of the corrosion and may be inform of impurities.
Let

$$\Gamma_a = \frac{1}{h_1} + \frac{2k}{a} + \frac{1}{h_c} + \frac{1}{h_2} + \xi(\mu)$$

$$T_a = u(t,a), T_b = u(t,b)$$

Then

$$\Gamma_a Q = u(t,a)$$
$$\Gamma_b Q = T_b = u(t,b)$$

It follows that

$$(\Gamma_{a^1} - \Gamma_a)Q = \frac{-2hk}{a^1(a^1+h)} Q = u(t,a) - u(t,a^1)$$

Therefore

$$Q = \frac{a^1(a^1+h)}{2hk}(u(t,a^1) - u(t,a))$$

This follows that:

$$h = \frac{\alpha}{\beta Q + \gamma}$$

Where

$$\alpha = \alpha^{12}, \beta = 2k, \ \gamma = -a^1(u(t,a^1) - u(t,a))$$

By dimensional analysis, we found that the life expectancy of the plant is related to h and the Corrosion rate A = $\frac{12w}{tnp}$ by the relation

$$T = \frac{Const}{hA}$$

Const = C is a dimensionless constant that can be determine experimentally, hence

$$T = \frac{tAPC}{12\alpha w}\ (\beta\alpha + \gamma) \qquad (*)$$

Acceptable working condition of the plant containing carbon and low alloy steel

Time (year)	Life expectancy
Completely satisfactory	$< 0.01(\beta + \gamma/\alpha)$
Use with caution	$< 0.03(\beta + \gamma/a)$
Use only for short expensive	$< 0.06(\beta + \gamma/\alpha)$
Completely unsatisfactory	$> 0.06(\beta + \gamma/\alpha)$

For high alloy steel, brass and aluminum

Time (year)	Life expectancy time
Completely satisfactory	$< 0.005(\beta + \gamma/\alpha)C$
Use with caution	$< 0.03(\beta + \gamma/\alpha)C$
Use only for short exposure	$< 0.03(\beta + \gamma/\alpha)C$
Completely unsatisfactory	$> 0.03(\beta + \gamma/\alpha)C$

Zhimz-Hoffman (ZH) Model

In this section, we proposed an adapted Zhimz - Hoffman's model that will take care of phase transition that arises as a result of corrosive fluid on the solid surface of the plant. This model is particularly useful in cracking process in the refinery; it is worthy of note to mention that corrosion of the surfaces into globules of impurities may cause catalytic poisoning of a petrol chemical plant.

The Zhimz - Hoffman's process is a typical non-linear parabolic partial differential equation whose solution exists in Ω X (0,T). In the absence of phase transition, that is, $\varphi = 0$, the equation reduces to the classical heat equation. The solution of Zhimz - Hoffman's model from analytic point of view is not generally easily obtainable.

The use of computer via numerical approach is highly recommended. This can be done through applications of the finite difference scheme, collocation method or the finite element method to the model. The advantage of the ZH method being its accuracy is highly guaranteed to some degree of freedom.

One striking thing about the ZH model is that it relates the relevant constants such as k, T and L that are in fact the properties of the surfaces of the material from which the plant is fabricated. These constants differ from material to material; the importance of the phase transition cannot be overemphasized.

We state the Zhimz - Hoffman's equation as

$$\frac{\partial u}{\partial t} + \frac{1}{2}\frac{\partial \phi}{\partial t} - K\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) = f$$

$$\tau \frac{\partial \phi}{\partial t} + \varepsilon^2 \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) = g(u) + 2u$$

$$u(x,0) = u_0(x),\ \varphi(x,0) = \varphi_0(x) \text{ as } \Omega \times \{0\}$$

Where

$g(\varphi) = a(x)\varphi + b(x)\varphi^2 - c(x)\varphi^3$; c^0, k, l, τ, ε^2

Are prescribed positive constants, u is the temperature the phase transition, K is conductivity, and l is the latent heat released or absorbed during phase transition ε measures sharpness of the free boundary and τ the surface tension.

The usefulness of the above model can be buttressed by the fact that, most heat generation problem in a corrosive media, it is often accompanied by phase changes. The problem can thus be modeled by well-known phase field model, which describes the phase transition between two different phases, e.g. solid and liquid. The Zhimz and Hoffman's adequately take care of this.

Using experimented data, i.e., computation of the temperature from Zhimz - Hoffman's using the computer, the time expectancy of the industrial plant can be computed, we utilize the formula in eqn. (*) for the computation. Different materials can be subjected to corrosion testing and isolate which material has the best expectancy life.

In practice, it has been found that material formed from high concentration of alloyed have greater life expectancy, although, such materials are expensive.

6. Galvanic Corrosion

Electrolysis has played a number of practical applications e.g. "isoelectric focusing" for separating large molecules of protein (Cho et al. (1994)) and in the hydrodimetrization of acrylonitrille for the nylon 6,6 industry (Tutty - Denualt (1994)). In this section, the idea of galvanic corrosion is discussed, even though, the discussion may not be as cuspicious as possible. This is an area where corrosion actually plays some useful industrial application.

In galvanic corrosion, dissimilar metals are placed in contact in an electrolyte, the corrosion rate of the anodic metal increases while that of cathode one decreases. This practice has had useful industrial galvanization applications (Chol et al, 1994; Babski et al, 1983; Sawille and Palusinski, 1986; Zlvenga and Egocheage, 1989).

Tutty - Denualt (1994) discussed the use of mediators in the galvanization process; we start our discussion with work of Choi and his associates:

They considered a single - dissociation - association reaction of the form

$$A^0 \leftrightarrow p\,B^{z_2} + qC^{z_3}$$

Where

A is the neutrally charged species and B and C are the positively and negatively charged species, respectively.

Hence, p and q are positive integers, and $z_1 = 0$, $z_2 > 0$, $z_3 < 0$ are the charges of A, B, C respectively.

Applications of laws conservation of charges and mass kinetics leads to the equations

$$pz_2 + qz_3 = 0$$
$$r = k^f u_1 - k^r u^p{}_2 u^p{}_3$$

Where

k^f and k^r are positive reaction rate constants and u_1, u_2, u_3 are the concentrations of A, B, C respectively.

We shall not go into discussion of the mathematical equation governing the Choi's model we note that the steady state equation for the electrolysis is given by

$$au^{11} - (bu + u^{p+q}) = f(x) - \beta$$

Where

$$a = \frac{d_2 d_3}{qd_2 + pd_3}(1 - \frac{z_2}{z_3}),\ b = \frac{d_2 d_3 k^f}{d_1(qd_2 + pd_3)}(1 - \frac{z_2}{z_3})$$

$$C = k^r(-\frac{z_2}{z_3})^2,\ f(x) = \int_0^x (\frac{k^f}{d_1}\int_0^y S(\varepsilon)d\varepsilon)dy$$

$$-\frac{k^f p d_3 x}{d_1}(d_1(qd_2 + pd_3)\int_0^1 sdy)$$

And

β is undetermined constant.

From their definitions, a, b, c are positive, and f is a known function. So that the boundary condition

$$u^1(0) = \alpha_0, u^1(1) = -a_1, u = u_2 \text{ is satisfied.}$$

Investigation of qualitative behaviour of above problem was found that the steady - state solution of equation is bounded above by max $u(x,\beta_2)$ in the interval [o,1]; and the parameter β_2 depends on the parameters of the model.

Using the following monotone iterative equation

$$au^{//}u''_{(n+1)} - bu_{(n+1)} - \Omega_n u_{(n+1)} = f(x) - \beta + C u^{p+\varepsilon}_{(n)} - \Omega_n u_{(n)}$$

Subject to boundary condition

$$u^1_{(n+1)}(0) = \alpha_0 \text{ and } u^1_{(n+1)}(1) = -\alpha_1$$

Where Ω_n is a constant such that

$$c(p+q)u^{p+q-1}_{(n)} < \Omega_n$$

The monotone iterative equation can be recast in the form

$$F(\beta) = g^{-1}{}_2(\frac{\gamma_2}{u(1,\beta)}) - g^{-1}{}_1(\frac{\gamma_1}{u(0,\beta)})$$

$$-\frac{pq}{z_2(qd_2 + pd_3)}\int_0^1 S(\varepsilon)d\varepsilon\int_0^1 \frac{1}{u(x,\beta)}dx$$

$$+\frac{d_2 - d_3}{z_2 d_2 - z_3 d_3}\log(\frac{u(1,\beta)}{u(0,\beta)}) = 0.$$

The numerical solution for the zero of the above equation can be found using the secant method as follows:

$$\beta_{n+1} = \beta_n - F(\beta_n)\frac{\beta_n - \beta_{n-1}}{F(\beta_n) - F(\beta_{n-1})} \qquad n = 2,3,...$$

We can find β, and hence $u(x,\beta_3)$ which can in turn is use to calculate $F(\beta_3)$, and β_4. $|\beta_{n+1} - \beta_n|$ is within a given tolerance limit.

The values of β_{n+1} and its corresponding $u(x,\beta_{n+1})$ give the numerical approximation of the method:

Δx	T_{o1}	N	B
0.01	10^{-4}	9	5.56
0.01	10^{-4}	8	1.91

One important application of Choi model is principle of isoelectric focusing. This method has biotechnological application; it is often required to separate protein molecules in a given mixture. A mixture containing protein molecules are placed buffer solutions contain two electrodes. A potentials difference produced at the ends of electrodes, at steady state, force the protein molecules to migrate and concentrate at various locations in the electrolyte. That is, get localized and it is refer to as isoelectric (focusing) point everywhere in the column.

7. Corrosion in a Gas Pipeline

In gas distribution line, the application of corrosion is not easily conceivable. If we assume the gas flow is corrosive and turbulent, we can obtain an expression for weight loss due to corrosion.

It has been shown that for a clean commercial pipe, the total annual operating cost is

$$C_f = \frac{H_p}{E} 4.13x\,10^{10} G^{2.84} \mu^{0.16} p^{-2} d^{-4.84}$$

Where

G = flow rate, kg/s
ρ = density, kg/m^3
μ = viscosity, MNm^{-2}
d = pipe diameter ,m
E = pump efficiency per cent/100
H = plant attainment hr/yr
If d_o is the diameter of the pipe before corrosion taken place and after corrosion it is d then,

from below diagram

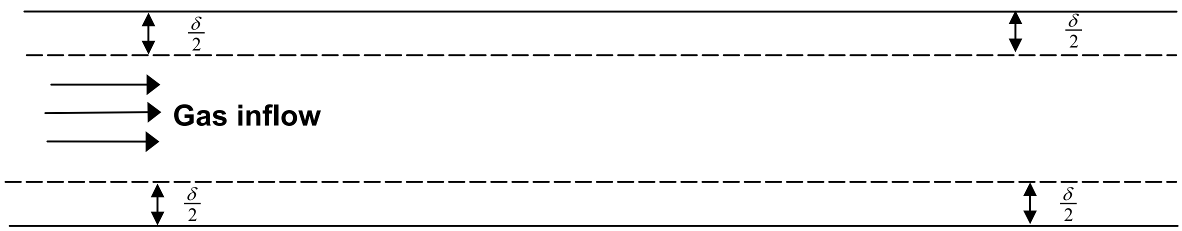


Fig 3: Corrosion in pipeline distribution

it follows that $d = d_0 - \delta$.

Now let

$$C_r = \frac{H^r{}_p}{E_r} 4.13 \times 10^{10} G^{2.84}{}_r \, \mu^{0.16}{}_r \, p^{-2}{}_r$$

And

$$C = \frac{H_p}{E} .4.13x\, 10^{10} G^{2.84} \mu^{0.16} p^{-2}$$

And also

$$C_f = Cd^{-4.84}$$

$$C_{fr} = C_r d_0{}^{-4.84}$$

Therefore

$$\frac{C_{fr}}{C_f} = \frac{C_r}{C}(1 - \frac{d}{d_0})^{-4.84}$$

Hence

$$\delta_0 = d_0(1 - (\frac{C}{C_r})(\frac{C_{f_r}}{C_f}))^{4.84} \approx d_0(1 - 4.84(\frac{C}{C_r})(\frac{C_{f_r}}{C_f}))$$

If $(\frac{C}{C_r})(\frac{C_{fr}}{C_f}) < 1$

The expression for δ_0 gives the corroded radius and the corroded surface lost being

$$\frac{\Pi(d_0^{\,2} - \delta_0^{\,2})}{4} L \;\; \rho = 2.84M(\frac{C}{C_r})(\frac{C_{fr}}{C_f})(1 + 2.42(\frac{C}{C_f})(\frac{C_{fr}}{C_f}))L \;\; \rho\, d_0$$

Where

L is the length of the pipe and ρ the density of the material from which is the pipe is made.
Thus

$$\frac{Md_0^2}{4\pi\rho d^2}\left[\frac{C}{C_r}(1 + 2.42\frac{C}{C_f}\frac{C_{fr}}{C_f}\right]L\rho d_0$$

v=1.071X10^4 a dimensionless constant. Using the above equations, life expectancy profile of various metals can be calculated when exposed to gaseous substances as

$$\frac{Md^2V}{4\pi\rho d^2}\left[(1 - \frac{d}{d_0})^{-4.84}(1 - 2.42(1 - \frac{d}{d_0})^{-4.84}\right]L\rho$$

Early Life Failure due to Corrosion

We can compute the reduction or elimination of the early-life failure (ELF)
Of the equipment caused by other forms of corrosion can be empirically derived from the Weibull distribution function given by

$$f(t) = \frac{\beta}{\lambda}(\frac{t}{\lambda})^{\beta-1}\exp(-(\frac{t}{\lambda})\beta$$

Where

$\beta =$ shape parameter

$\lambda =$ Characteristics life - time of the equipment

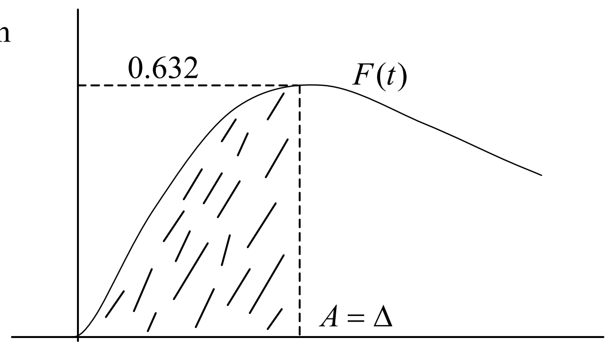
t= time.

The values for β and λ can be determined experimentally for various materials. For example if $\beta = 1$ and $\lambda = 1000$, the ELP curve is (fig.4)

The cumulative failure can be obtain from

$$F(t) = \int f(t)dt = 1 - \exp(-(\frac{t}{\lambda})\beta)$$

If $t = \lambda, F(t) = 0.632$



ELP curve

i.e.63.2% life span of the equipment has expired .At this state the equipment is fatigued and most likely prone to corrosion.

Finally, it must be emphasized that the actual statistical distribution to the most ideal distribution function governing the failure rate of various equipment is a thing of serious consideration and should be subjected to further enquiry.

Acknowledgements

The first author wishes to express his gratitude to Professor S.O.Ale the Director of NMC for involving him in mathematical modeling group of Environmental Problems of the National Mathematical Centre, Abuja Nigeria while on sabbatical leave at the centre. He is also grateful to Professor Asere A.A., for suggesting application of mathematical modeling to corrosion problem. He is grateful to the Industrial Training Funds (ITF) and the ATBU Linkage Team for co-opting him into study of industrial problems in Nigeria.

References

[1] ALE S.O. (1981)
"Curriculum development in modelling process". Trieste, Italy, International Centre for Theoretical Physics occasional publication,1981 ,SMR/99 - 42.

[2] ALE S.O. (1986)

"Encouraging the Teaching of mathematical modelling in Nigerian schools."
Nigerian Education Forum 9(2), 185 – 191,1986.

[3] Babski M.Yu., Zhukov and Yodovich V.
"Mathematical theory of Electrophoresis". Applications to methods of Fractional Biophysics,1983.

[4] Beddling S.
"Concerning an integral arising in the study of laser - Drilling Equation". International Journal of Science Technology, Vol. 25, NOS, 609 – 672,1994.

[5] Berry John
"Mathematical modeling: A Source Book of Case Studies". Edited by I.D Huntley and D.J.G James, Oxford University Press London, pp 81 – 96, 1990.

[6] Carslaw, H,S and Jaeger J.C.
"Conduction of heat in solids". 2nd edition. Oxford University Press,1973.

[7] Choi Y.S, Roger Liu and Xun Yu
"Electrolysis and Isoelectric Focal". Institute of Mathematics and Application Journal of Applied Mathematics. 52, 105 – 122,1994.

[8] Morton Daniel
"Mathematically Speaking". New York: Harcourt Brace, Jovanovich, Inc,1980.

[9] James Verghese (gen.,editor)
"Terpene chemistry". Tata- McGraw - Hill publishing company. New – Delhi,1980.

[10] Jones G.D.
"Chemistry and Industry: Applications of Basic principles in Research and process Development". Clarendon press, oxford,1967.

[11] Zhimz and Hoffman
Institute of Mathematics and Applications Journal Numerical analysis. 14, 243 – 255,1994.

[12] Tutty and Denualt
"Second-Order Kinetics for EC Reactions at Special Micro Electrode". Institute of Mathematics and Application Journal of Applied Mathematics. 53,95 – 109,1994.